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Vitaly Kaganov

A unified capital asset pricing model

Ujednolicony model wyceny aktywów kapitalowyh

Doctoral thesis

Supervisor: dr hab. Katarzyna Perez, associate prof. of PUEB

Department of Monetary Policy and Financial Markets
Doctoral Seminars in English

Poznań 2018

Content

Spis treści

Introduction

In 1965 Fama (1965) established the *Efficient Market Hypothesis* (*EMH*) which main idea is to demonstrate that past information has no influence on the current price formation and the *random walk* makes the prices unpredictable. The *EMH* argues that market prices (*P*) are always on its *fair value*, i.e. it does not under/overpriced and equals the fundamental price (*F*): $P = F$. Later, Shiller (1981) discovers large price fluctuations compared to the fundamental prices of the *EMH* theory, known as the *volatility puzzle*. Leaning on the *Prospect Theory* of Kahneman and Tversky (1979), Shiller (1981) argues that the source of such phenomenon lies in the non–rational human nature. Psychological or sociological background of an individual may motivate him/her to make non–rational decisions relatively to his/her investment, which prevents from an individual to reach the fundamental price, i.e. this implies that market observable price (*P*) does not equal the fundamental price (*F*): $P \neq F$.

Black (1986) argues that the reason for volatility phenomenon is some *noise* which exists all over the markets and creates a divergence from the fundamental value as a result of environmental circumstances, preventing normal distribution of information. *Noise traders* obtain lots of information, which comes out from technical analysts, economic consultants and stockbrokers, falsely believing this information is useful to predict the future prices of risky securities. Following Black (1986), it implies that market observable price (*P*) is the sum of fundamental price (*F*) and noise (*N*): $P = F + N$. In contrast, behavioral explanations and models are based on a specific psychological bias. Most common behavioral biases were generalized by Szyszka (2009) within his *Generalized Behavioral Model* (*GBM*). According to the model of Szyszka (2009), the market observable price (*P*) equals the sum of fundamental price (*F*) and behavioral component (*B*): $P = F + B$.

The main question, the literature tries to answer, is how investors react to new information, when 2 variants are possible — rational–base reaction or behavioral reaction. It is also possible to understand that the deviation from the fundamental price is either noise or behavioral component, i.e. $N = B$. Conducting the literature review, I suggest that all types of the investors are simultaneously present on the market. Hence, the market observable price (*P*) should be equal to the sum of fundamental price (*F*) and nonfundamental price (*NF*), which is sum of noise (*N*) and behavioral component (*B*):

$$P = F + (N + B).$$

My suggestion has a potential to create a platform for one integrated and solid financial theory. I believe that integration of the best achievements from both theories will lead to better results and to more accurate financial reality description.

The main goal of my PhD thesis is to propose and to test a new approach which combines normative and behavioral approaches in one Unified Capital Asset Pricing Model. The creation of the unified model is motivated by *unification* of traditional and behavioral approaches that in turn leads to *universality*, where rational–based and behavioral–based investors use the same model to determine daily returns. From here the sub–goals are:

1. Presenting normative and behavioral approaches to asset pricing and comparing them.
2. Describing and comparing empirical findings on non–fundamental component as well as on normative, behavioral and unified models.
3. Proposing and testing the mechanism allowing capital pricing assets, which can be used in investment decision process.
4. Comparing the proposed model to existing models and checking whether it has more predictive power than those models.

There are 3 hypotheses derived from these goals:

H1: *Deviation components hypothesis* — the deviation from the fundamental price indeed can be explained in terms of noise and behavioral components, i.e. in terms of *Technical Analysis* index and *Sentiment Indicators*.

H2: *Explanatory performance hypothesis* — the Unified Capital Asset Pricing Model has a better explanatory power of the deviation from the fundamental price than traditional or behavioral approaches separately, which is expressed in higher R^2. It is obtained when $R^2 \geq 0.5$ at least for the fully integrated regressions.

H3: *Significance hypothesis* — the components of the Unified Capital Asset Pricing Model are statistically significant.

The methodology of this study is closely related to the existed literature with several modifications and applied within 4 stages and the data for 2 stock markets of the US and Israel in 2001–2017 is applied. At the 1st stage all the goals, hypotheses and variables are defined. As the unified model assumes 3 powers are involved in the explaining daily returns, every single power accepts its unique measure. The fundamental returns are described by the *CAPM* model for the Israeli market and by Fama–French five–factor model for the US market, which are determined by rational investors as the *EMH* suggests. The noise is expressed in terms of *Technical Analysis* and the variables are evaluated according the methodology of Neely et al (2014) with several modifications. The behavioral component is expressed by *Sentiment Indicators* and evaluated according the methodology of Sadaqat and Batt (2016) and Yang and Zhou (2015) also with several modifications, assuming it is determined by investors with psychological biases. The evaluation method for all models is the *Principal Component Analysis* (*PCA*) and estimation method is *OLS*. Both these methods are widely accepted in the literature.

At the 2nd stage the ability of chosen technical and sentiment variables to explain daily returns is examined and compared. The comparison is made among all relevant models (including three and fife-factor models) in both US and Israeli markets. At the 3rd stage analysis technical, sentiment and unified predictors are created through the principal component. Further, explanation power with coefficient pattern and significance of all predictors for daily returns are compared. At the last, 4th stage, the integration of relevant fundamental component with predictors, derived from *PCA*, is applied and further comparison between all the models is done. Moreover, in this stage hypothetical alternative model, where all the components are integrated directly after the *PCA* with the fundamental component, is presented and its results are compared to all other models. The analysis ends with the conclusions.

There are several points of this PhD thesis which are the contributions to the literature. Two different, sometimes contradicting theories exist side by side describing the same subject is a paradoxical situation. Probably if one theory could be better than the other, only one would survive. In my PhD thesis I introduce a Unified Capital Asset Pricing Model in order to fill the gap between normative and behavioral theories. The suggested model should answer the question whether it is possible to improve the performance of every single model separately by unifying both in one model. Theoretically, the unified model surpasses every single existed model in every possible parameter regarding the daily returns. Indeed, the findings demonstrate that unified model may contribute to explanation of both normative and behavioral approaches on the background of stable coefficients. In this situation the unified model improves the explanatory ability significantly in the US and Israeli markets. The integration of the fundamental factors leads to noticeable increase in the performance of all models in both markets. However, only the unified model has the most prominent achievement. The value of R^2 of the united model can easely exceed 0.5 and in vast majority of cases it exceeds 0.55 while for other models, including five–factor model, it hardly exceeds 0.5. Even in the case of the alternative model, where technical and sentiment predictors, retained from the principal component analysis, are directly integrated with fundamental factors into one model, the unified model still have a better performance. The unified model is able to improve explanatory power of the alternative model even more than those of the five–factor model regarding to the three–factor model.

In addition, it was found that all the models demonstrate similar coefficient patterns and predictive ability on the US and the Israeli markets, which may indicate that such phenomenon appears internationally. If so, the unified model is even more universal than it was assumed at the beginning. For this reason, such phenomenon deserves to be investigated deeper.

The literature suggests to involve only 2 powers in the determination of returns. The unified model suggests 3 powers that include more types of investors what should describe the financial reality better than previous approaches. Indeed, it is found that unified model has the ability to improve the performance of the existed models significantly. The Unified Capital Asset Pricing Model is actually a first attempt to unify main financial theories into one solid platform. According to the results obtained in the study there is a high potential to achieve improvement in performance of existed models with subsequent creation of only one financial theory, describing the financial reality the most appropriate way.

The thesis contains of three chapters. Chapter 1 provides a theoretical background for different capital asset pricing theories/approaches, including leading models in each one of the fields. Chapter 2 contains most important and influential studies on the theoretical models that are introduced in Chapter 1. Chapter 3 provides full description and theoretical background of the proposed Unified Capital Asset Pricing Model, including tests and comparison of its results with those of existed theories and models.

Chapter 1

Theoretical background for capital asset pricing

1.1. History and idea of capital asset pricing

The question of saving financial resources remains vital in any era. Searching for the answer leads, in general, to two questions. First question is about the decision of how to save and the second is how to keep the actual purchasing power of the savings in the future. The future is uncertain, hence before the individuals raise a question of risks and subsequent loss of their savings. In order to reduce the negative influences of the risks, it is necessary to reduce the uncertainty or to perform a diversification or both. Predictability in some degree of confidence may reduce the uncertainty, hence many researchers try to develop sufficient models to forecast asset prices. Such models answer the question of future prices but also the question of the optimal saving spending.

In this chapter I present the fundamentals for the capital asset pricing theories and for the technical analysis. I concentrate on the historical and theoretical background as well as on the description of the main models for classical and behavioral approaches. At the end of the chapter the comparison between two main approaches is also done.

Capital Asset Pricing Theories

The modern capital asset pricing theories are based on mean–variance analysis as shown on Figure 1. Further evolution of those theories is divided on classical (normative) and behavioral finance.

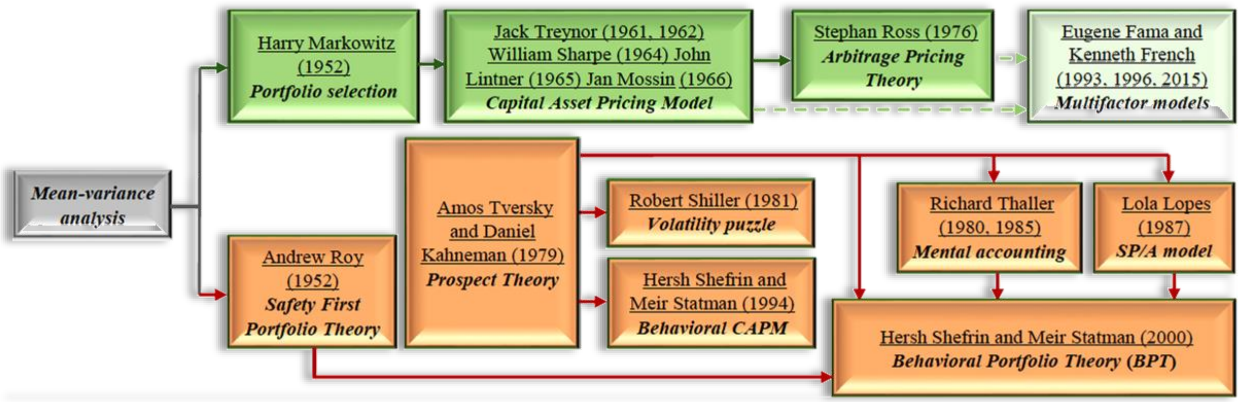


Figure 1. Evolution of capital asset pricing theories
Source: Own work

In **1952, Markowitz** introduces a publication about the optimal portfolio choice, which is the *Modern Portfolio Theory* (*MPT*) based on a mean–variance analysis. The theory focuses on choosing an efficient strategy for *risk diversification* and justifies expediency of the diversification. The economic intuition behind the risk diversification is natural and understandable for individuals when the idea exists a very long time, but before Markowitz (1952) the diversification strategies were naive and made by a guess. He explains which assets are preferably included in a portfolio due to a perfect negative correlation with each other.

In his work, Markowitz (1952) makes a connection between risk, measured in terms of volatility and returns. He also argues that it is possible to reduce the risks without any harm to the *Rate of Return* (*ROR*). He defines the set of points (portfolios) that are the most efficient due to their maximal diversification. The set creates a line of *efficient frontier*. Markowitz (1952) involves the *risk–free* asset assuming that combining a portfolio on the *efficient frontier* with the *risk–free* asset in a different proportion should give a better result than a portfolio which contains only risky assets. The *MPT* has a great contribution since it revolutionized the approach to finance as a whole, turning it to analytical. The *MPT* lays in the very foundation of further ***Capital Asset Pricing Model* (*CAPM*)** and of whole neoclassical capital asset pricing.

The *CAPM* was independently and separately introduced by 4 researchers: **Treynor (1961, 1962)**, **Sharpe (1964)** and **Lintner (1965)** as well as by **Mossin (1966)** and it is a straight logical evolution of the Markowitz' *MPT*. The main idea of the model is that the assets are correlated not only with each other but also with the market. Such correlation is the only factor that determines the expected returns, signed as β coefficient, which plays a role of a risk premium. The *CAPM* takes the idea of Markowitz (1952) about measuring risks in terms of volatility further and argues that the market volatility is probably the risk, which should be compensated with an appropriate level of *ROR* through correlation with the market itself. If so, then the asset price that brings the risk–return pair to its optimal level is the equilibrium and called a *fundamental price*. The model is simple to intuitive understanding of every, even naive, investor that turned it to popular among both: the practitioners and the academia. The *CAPM* is the first fundamental capital pricing model that is still in use even in present days.

The tests of the *CAPM* revealed that it does not fit to reality, which will be discussed in detail in Chapter 2. **Ross (1976)** fixes the misspoints of the *CAPM* and proposes a multifactor model, which is the ***Arbitrage Pricing Theory* (*APT*)**. In this model, it is assumed that the correlation only with the market is insufficient since the market itself is depended on macroeconomic environment. If so, there a specter of risks that influence the expected return where every one of them has its own correlation through its own *beta* coefficient.

In the parallel with Markowitz, **Roy (1952)** introduces another optimal portfolio preference. This publication becomes less popular than the *MPT* among the normative economists. Nevertheless, his work gave a push to behavioral finance and became known as ***Safety–First Portfolio Theory*** (***SFPT***). Roy (1952) assumes that there is a probability that may cause an overall collapse to an individual that he calls the probability of ruin. Such probability should be minimized that is equivalent to minimizing the number of standard deviations in which wealth level s lies below the portfolio mean (μ_p) under normal distribution.

The difference of two theories lays in their motives. Markowitz (1952) keeps a place to an investor to decide about a desirable portfolio within the *efficient frontier*. Roy (1952) points the exact portfolio that an investor should chose and then constructs his *CML*. That is because Roy (1952) sees the world as a set of risks that some of them may cause total disaster to an individual. For this reason, an individual should prepare himself for such scenario by diversification among accessible assets and by choosing assets the manner that exceeds the probability of a disaster occurrence. This interesting interpretation of mean–variance analysis rose, as Roy (1952) believes, from unexplainable but observable behavior of individuals, driven by psychological aspects, though he is still leaning on an expected utility and normal distribution of portfolio returns as the normative theory suggests.

In **1979**, **Kahneman** and **Tversky** introduce their ***Prospect Theory***. They were interested to investigate how individuals make their decision facing the uncertainty and how they calculate a subjective probability to make their judgment of uncertainty comparable. For this reason, different prospects introduced to the participants and the results show that the individuals have a number of limitations. This real case study proves that the individuals have a different fundament for the uncertainty judgment than the one suggested by classical theory. It also shows how psychological biases may affect a way of how an individual makes his decision facing the uncertainty.

The theory describes a decision processes and tries to model real–life choices, rather than optimal decision as normative models do. According to the theory, the individuals make their decision leaning on the potential value of losses and gains with its respective probability rather than on the final real outcome.

Since the *Prospect Theory* was introduced, the monopole of the classic paradigms was over and a new cluster of financial thinking, which is the *behavioral finance*, was created. This is the alternative approach to the classical normative theories raised as a result of the studies of the researchers like **Shiller (1981)** who observed ***volatility puzzle*** and explained it in the terms of investors' non–rationality; **Thaler (1980, 1985)** who explained that money may have different value in an investor's mind, which is known as ***mental accounting*** as well as **Shefrin** and **Statman (1994)** who made the behavioral extension to the *CAPM*. The heart of the *behavioral finance* is the assumption that the human beings are not necessarily rational in the sense of the traditional concept of *Homo economicus*. Therefore, individuals make their decisions involving non–economic factors.

Once the principals of the *behavioral finance* were established, the behaviorists turned to look after specific biases that may affect an individual's investing behavior and cause a price to deviate from its fundamental value. Here every specific bias lay in a basis of a behavioral asset pricing model:

- Barberis, Shleifer, and Vishny (1998) propose a model of investors' sentiment;
- Daniel, Hirshleifer, and Subrahmanyam (1998) assume the overconfidence of the investors;
- Hong and Stein (1997, 1999) assume that the information distribution is gradual and hence is unequal.

Lopes (1987), based on the *Prospect Theory*, publicizes a decision–making model under uncertainty, which is the ***SP/A* model**. She introduces a dual–sided problem that an individual should solve simultaneously. Lopes (1987) introduces a general individual descriptive problem, though she does not suggest any sufficient solution for it. According to the theory, an individual is about to solve how to secure himself from the subsequent *fear* additionally to achievement of aspiration from the subsequent *hope*. Possibly because of the fact that the problem should be resolved simultaneously, a solution was not introduced. The qualitative lesson from the *SP/A* model is that an individual should establish a portfolio in which he has a full control on the emotions of *fear* and *hope*, so that neither of them is dominant. Although, this intuition of dual choice problem is in the consideration of different behavioral researches under different definitions of the same variables.

The *SP/A* model, *Prospect Theory* and *mental accounting* with Roy's safety–first model allowed **Shefrin** and **Statman (2000)** to develop the ***Behavioral Portfolio Theory*** **(*BPT*)** as a response to the *MPT*. In their model they propose that every single investor is not interesting only in one investing portfolio, but he is likely to spread his investment between a number of portfolios, for each one its own goal and horizon. The model of Shefrin and Statman (2000) is a sort of the mean–variance analysis, looking towards the behavioral biases of the investors.

The generalization of all psychologically biased models was made by **Szyszka (2009)**. After a massive survey of behavioral literature (Szyszka, 2007), he realizes that only three psychological biases are widely common. For this reason, he decides to incorporate those three biases into a model, which is named ***Generalized Behavioral Model*** **(*GBM*)** today. The *GBM* is successful in explaining many of the observed market anomalies.

Technical Analysis

Aside to the development of the fundamental theories, the literature discusses another approach of forecasting the direction and trend of the stock prices movement with different principles, which is the Technical Analysis (see Figure 2).

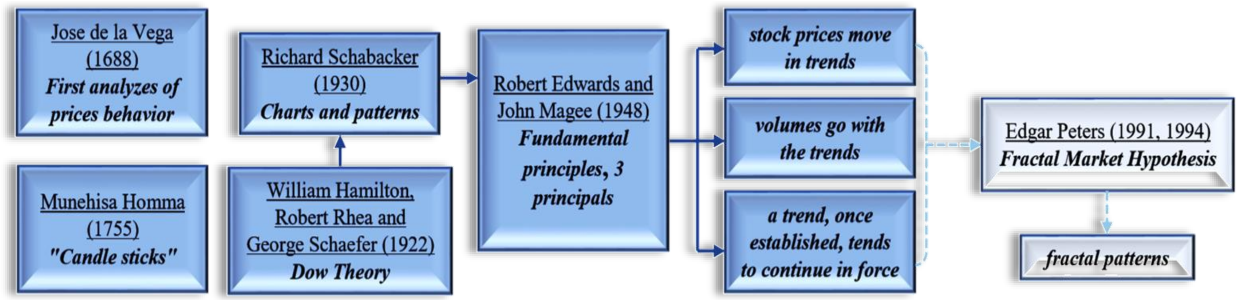


Figure 2. Technical Analysis development
Source: Own work

There is some evidence that several technical tools have been used to determine the stock prices already in the 17th century. The successful Jewish merchant from Cordoba, Jose de la Vega (1688) wrote a book "*Confusion of Confusions*", which is the first book ever devoted to the stock market. In the book, he analyzes the prices behavior and introduces some techniques to forecast it. He describes Amsterdam Stock Exchange and different market tools like options of calls, puts and pools. In addition, he describes some speculative strategies.

Another example of early *Technical Analysis* can be found in Japan. Munehisa Homma (1755) was the Japanese rice merchant who developed a unique trade technique that is in use even in the present days and known as the ***Candlesticks Charts***. In his book, Homma (1755) claims that psychological aspects of the individuals are crucial. Emotional background composites the bear or the bull market. He notes that recognizing this can enable one to take a position against the market.

In the western world, the technique was introduced by Nison (1991) that later claimed that the technique was actually created in 1800 and indeed was used on the Japanese rice market to forecast the prices. Candle is an abstract name for a figure that is created every trading day by open, close, daily high and daily low prices. Conditionally to the market forces a candle might be colored by black or white. Different forms of the following candles indicate some market positions that help a decision maker to "strike the market".

Though the existence of some old techniques of the *Technical Analysis* have been known since the 17th and 18th centuries, the main part of the theory development started in early years of 20th century. Today it is ***Dow Theory*** after the name of the American journalist, co–founder of Dow–Jones index and *The Wall Street Journal*. This theory was possible through the financial data accumulation in the hands of Dow. The theory with its 6 postulates, which is the basis for a decision making, was described in 1922 by the journal co–editors Hamilton, Rhea and Schaefer as a compilation of a momentum strategy. Those are:

(P1) – there are three types of trends:
long (bullish/bearish);
medium swing that comes after the major movement;
short swing which represents the market reaction to a trend turn;

(P2) – there are three phases to a market trend:
(1) first when the smart investors recognize the trend and buy/sell against the market;
(2) second when the trend becomes obvious and the active traders have started to buy/sell in large amounts, influencing the price;
(3) third when the whole market understands the trend and moves like it;

(P3) – all the information is already priced;

(P4) – all the indices should be correlated with each other when a trend is changing through the timing may differ;

(P5) – a trend should be confirmed by a volume;

(P6) – a trend is valid until a clear signal of its end.

Schabacker (2005), the editor for Forbes magazine and continuator of the work of Hamilton and Dow, concentrates in his book on a comparison between the fundamental indicators and the technical ones. He also pays attention to the charts and patterns. He realizes that whatever a significant action appeared in the average, is a consequence of a similar action in some of the stock prices, making up the average. Schabacker was a pioneer of collecting, organizing, and systematizing the technical tools. He also introduced new technical indicators in the charts of stocks; indicators of a type that would ordinarily be absorbed or smothered in the averages, and hence, not visible or useful to Dow theorists (Edwards & Magee, 1948).

The seminal work of the theory was done by the Schabacker's nephew Edwards and Magee (1948). In their book "*Technical Analysis of Stock Trends*", they conduct the fundamental principles and the goals of the analysis. They revise the previous works in the area that are mostly concentrate on charts and trend recognition because of low computing power, and found it as good enough, but with their own extensions of new technical methods. They postulate only three principles:

(P1) – stock prices move in trends;

(P2) – volumes go with the trends;

(P3) – a trend, once established, tends to continue in force.

The fundamentals of the analysis are as follows: all the relevant information is reflected in the prices and the past is a better indicator for the future, rather than external economic factors. The past reflects behavioral situations and how the individuals really reacted to it. If so, it is reasonable to assume that in similar situations the individuals will act the similar way as in the past. *Technical Analysis* believes that investors collectively repeat the behavior of the investors that preceded them, which allows predicting the prices behavior. The goal of the analysis is to identify, using the technical tools, a trend or a swing where all the market should move and by this way to earn an advantage before the market, i.e. to ride the trend. This is the main difference with the fundamental theories that try to determine a price.

The theory became demanded and popular again in late 80s and middle 90s of 20th century with the evolution of computer technologies. Computers allowed the location of the important points in the charts and the analysis of the data much faster. Computer technologies helped to develop new analysis tools and techniques and new wave of publications swept the shelves in the forms of guides and manuals.

In the mid–90s, *Technical Analysis* pushed the idea of the ***Fractal Market Hypothesis*** **(*FMH*)** introduced by Peters (1991, 1994). Fractal is a functionally described geometric form that is a replica of a larger same–form object, for example a tree or a leaf. This means that there is an object that contains smaller parts that geometrically have the same shape. In the sense of financial markets, a price changing, sometimes, repeats itself in a form of patterns that allows assuming like repeating is a natural price behavior.

First, it was discovered by Mandelbrot (1963) in 2 effects: *Noah effect* is the sudden discontinuous price changes and *Joseph effect* when a price can stand for a short period yet suddenly change afterwards. Both effects violate the assumption of assets' prices normal distribution. To describe his results, Mandelbrot (1963) uses the ***Chaos Theory***.

The *Chaos Theory* is crucial for understanding the *FMH*. Though first attempts of Poincare (1892, 1893, 1899) to analyze mathematically the behavior of the dynamical systems, it was Lorenz (1963) that pioneers and develops the idea. Lorenz (1963) accidentally found that some systems are extremely sensitive to initial conditions. A miserable change in very short–terms may lead to significant changes in long–terms, making the dynamics mostly unpredictable. He begins to investigate the phenomenon and concludes that such system looks chaotic at first glance, but stem under some functionally described order. The **chaotic functions** are complicated for solving and the mathematics behind the *Chaos Theory* is not fully understood.

The theory tries to build a testable environment and uses complexed mathematics to support the hypothesis. The *FMH* believes that with additional development of the chaotic mathematics, the prices can be exactly predictable. It is established on three assumptions:

(A1) The investors can be rational or irrational in their decisions. What is more important, the investors are deviated in 2 groups depending on their investing horizon:

- the group of short–terms investors;
- the group of long–terms investors;

when an investment horizon is defined as the amount of time one plans to hold his money as an investment. In a case when the investors swing their strategy (becomes only one investors group), the market may crash by losing its liquidity. When another group is missing, no trading is possible;

(A2) The prices change on a basis of information that is relevant and meaningful to a certain investors' group. Therefore, an equilibrium price does not exist because investors value their investments differently and existence of the irrational investors automatically drives prices out from its fundamental value;

(A3) The *FMH* believes that there is short–terms stochastic process but long–terms deterministic process.

Even though the *Technical Analysis* is popular among investors, it suffers from hard criticism. The normative theories cannot accept the *Technical Analysis* despite some similarity of its views, because the basis is fundamentally different. The question of the predictability for the normativists is still open and has a different origin. The past cannot predict the future, even if some connection may be found it is not obvious that the behavior may be repeated. There is no hard proof that the *Technical Analysis* works in the mathematical or statistical sense. The main criticism is around the testability of the patterns.

1.2. Normative theories of capital asset pricing

The roots of normative theories of capital assets pricing are in the foundation of economics as a science itself (see Figure 3). Smith (1757, 1776) and his contemporaries describe the philosophical aspects of the concepts for liberal economics that became familiar with **classical economics**. In his books, Smith (1757, 1776) formulates the academic thoughts of his era. Some of them were progressively new and some were paraphrasing or extending exiting principles. In his famous books "The Theory of Moral Sentiments" (1757) and "An Inquiry into the Nature and Causes of the Wealth of Nations" (1776) (or simply "The Wealth of Nations"), he discusses the topic of free economics and proposes some postulates of new principals. Even in the present days some of Smith’s principles are still valid.

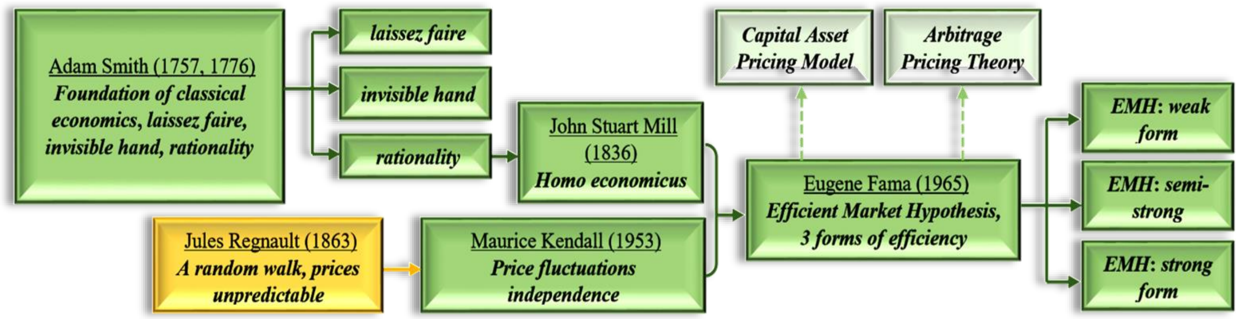


Figure 3. Background for normative theories
Source: Own work

In particular, Adam Smith adopts the term of ***laissez faire*** from the exiting in his days understanding of the economics and further develops it. He uses a term of ***invisible hand*** — an ability of a market to regulate itself by the inner power that comes out from the behavior of the individuals. The individuals intuitively understand the market mechanisms and they compose the market in such way, allowing them to reach **equilibrium** — a special condition where all parts of the market are balanced.

Smith (1757, 1776) postulates that all individuals should be driven by their own egoism. Egoism dictates the best for an individual and if all individuals may obtain their best, the overall social wealth should reach its maximum and hence desirable level. At this level, minimal poorness can be reached and will be necessary to obtain if an egoistic individual is free to act. A struggle of the individuals in the free market to obtain their possible maximum wealth should bring the individuals to a self–interested competition that would tend to benefit society as a whole by keeping prices low, while still building in an incentive for a wide variety of goods and services. As well as the individuals maximize their wealth, as Smith (1757, 1776) argues, the firms maximize their profit. He assumes that such concept is right relatively to both: an individual as a single unit or a whole nation as a single organism (in the sense of micro/macroeconomics).

In order to make this economic model to work, first, an economic agent must be defined. However, the egoism is not the only power that motivates the individuals, it is also the happiness. Smith (1757, 1776) describes the **agents as rational individuals**. Rationality, the fundamental term of all normative theories, is a specific behavior where an individual has some inner consistent clear and complete preference system, allowing him to rate all the products relatively to its usefulness that may be described by a utility function as a mathematical expression. An individual should prefer more than less and he prefers to enjoy than to suffer but to have less loss then more in reverse look. A rational man is motivated by facts and reasons to act and chooses to perform the action with the optimal expected outcome. Emotions are not rational and hence, are not taken into account during economic decisions of the individuals, because if they are, the optimal decision cannot be reached.

Later, in the 19th century, Mill (1836) extends the definition of rationality based on the Smith's concept and defines a term of *Homo economicus*. *Homo economicus* is a consistently rational, self–interested and labor–averse individual. The goals of such man are very specific and predetermined while he seeks to obtain it in greatest extent with the minimal possible costs. This definition is in use as approximation of *Homo Sapiens* and allows modeling economic behavior of human.

In the first half of 20th century the economics enriched with a solid theoretical basis and normative mathematical analytical tools. All this created a **neoclassical economical approach** which is recognized today as a *mainstream school*. The neoclassical economists postulate three principles that are the fundament of all in the economy. They are as follows:

(P1) – people have clear and complete preference system between outcomes that can be measured and translated to a value; the values have been produced with a utility function and comparable;

(P2) – all individuals maximize their wealth and all firms maximize their profits;

(P3) – the individuals make their decision according to expected utility and full and relevant information.

Neoclassical economists believe that the rationality rules all over and the individuals have accesses to all relevant information that is the very center of their decision. If so, it is reasonable to assume that formation of demand and supply has to be created with the same principles and talking about a stock market, the information is already priced within the stocks.

First determination of stock prices variability was made by Regnault in 1863. He argues that the stock prices evolve according to a **random walk** and hence, are unpredictable. He was the first who used statistics and probabilities to determine the stock prices. Further popularization of the idea of random prices movements refers to Kendall (1953). The theory states that stock price fluctuations are independent of each other and have the same probability distribution, but that over a period of time, prices maintain an upward trend. Past movement or direction of the price of a stock or overall market cannot be used to predict its future movement.

In 1965, in his doctoral thesis, Fama introduces the ***Efficient Market Hypothesis*** **(*EMH*)** based on the neoclassical principles. The *EMH* considers the agents as super rational individuals that have full accesses to all possible relevant information and properly use it in an extremely short time, where a rational agent is the one who wishes to maximize his expected return for a given level of risk. Subsequently, all the news are rapidly and fully reflected in the investment prices as it become known. Due to a short time pricing, competition drives all the information into the price quickly and there is no possibility to make any profit from the information in the long–run terms. This means that the assets are always traded on their **fair value**, i.e. they are not under/overpriced.

The available information is divided into 2 types:

– the first type is the information that is reasonably inferred, which means that prices will reflect beliefs of the market before the event actually occurs;

- the other type is all available information, which includes past information, current information, and announcements of future events.

The market price is not required to shift instantaneously or adjust to the perfect price following the release of new information. After an announcement is released, the price only must change quickly to an ''unbiased estimate of the final equilibrium price". The final equilibrium price will be reached after investors decide the new information’s relevance on the stock price.

There are three forms of the *EMH*: *weak*, *semi–strong* and *strong*.

- The ***weak form*** describes a situation where all available past and historical information is already priced by the market and is reflected in the stock prices. If so, the past information has no influence on future prices, hence, it is impossible to make any excessive returns in the future leaning on the historical data and knowledge. This means that any *Technical Analysis* tool is useless. Future price movements are determined exclusively by the information which is not contained in the price series. Therefore, the prices changes must follow a *random walk* and consequently, must be unpredictable. Although, the insiders that hold information before it is released to the market may obtain some abnormal returns above the average for a very short period.
- The ***semi–strong form*** describes a situation where new public available information is rapidly incorporated into the stock price. This implies that an investor cannot act on new public information and expect to earn any excessive returns. This time, neither the technical nor the fundamental analyses are able to produce any excessively returns above the predicted average.
- The ***strong form*** describes a situation where the stock prices fully reflect all public or nonpublic information, which means that even insiders cannot make abnormal profits in the market. The information about future events is also properly priced.

The *EMH* implies that no individual can benefit from the market consistently because stock prices follow a *random walk* and cannot be anticipated or be a basis for extra profit. If someone does outperform the market, it is only through luck or by a statistical chance. This means that a portfolio manager will not succeed to compose a better portfolio then those that were created by a blind random selection. The answer lies not in the returns of the chosen stocks, but in the risk of the chosen portfolio. If the market is efficient, portfolio managers are still able to obtain the appropriate level of diversification of the portfolio, they are able to obtain a higher return for a given risk level. This should eliminate firm specific risk and leave the portfolio only with systematic risk.

Also, it is important to understand that the *EMH*, assuming investors rationality, does not exclude the existence of non–rational individuals. Their activity may be seen as distortion of a pricing process that may create a disparity in the prices. As the theory suggests, due to prominent majority of the rational investors, such disparity will be closed quickly because rational investors are able to recognize it and act to obtain an extra profit until complete disparity closing.

The underlying principal of the *EMH* is established through the ***Central Limit Theorem*** **(*CLT*)**. The *CLT* states that as sample independent random variables are approaching infinity, the probability function is approaching the normal distribution curve. From this concept, the *EMH* assumes that market changes are random and if the market changes are plotted over a period of time they should construct the normal distribution curve. This way the *CLT* can be applied to historical data in order to find a correlation between the *EMH* and the observed financial market from which the data is taken.

Samuelson (1965) publishes a proof of prices random–walk behavior if a market holds the *EMH*, which is acceptable theoretical support of the theory of Fama (1965). Further, Fama (1970) publishes a review of both the theory and the empirical evidence for the *EMH*. His paper claims that the stock market holds **the micro efficiency**, but **not the macro efficiency**. Samuelson (1998) was sharing such opinion arguing the *EMH* is more suitable with individual stocks rather than with the aggregate stock market. Additional strong support of the random walk is issued by Malkiel (1973) in his influential book "***A Random Walk Down Wall Street*"**.

The *EMH* describes a capital market structure in the manner of normative vision and successfully integrated into existed normative approaches. Behind almost every normative capital asset pricing model stands the assumption that the *EMH* is valid. The *CAPM*, *APT* and further extensions of the *CAPM*, like Merton's (1973) *ICAPM*, accept the *EMH* as a starting point. Today, the concept of the *EMH* and its assumptions provide a solid platform and paradigm for modern normative economists.

In the following parts, next important and fundamental theories will be discussed:

- Expected Utility Theory (*EUT*) of von Neuman and Morgenstern (1944, 1953), which is a normative basis for decision making process under uncertainty.

- Modern Portfolio Theory (*MPT*) of Markowitz (1952), which is the normative basis for optimal investing choice.
- Capital Asset Pricing Market (*CAPM*) of Treynor (1961, 1962), Sharpe (1964), Lintner (1965) and Mossin (1966), which is the normative equilibrium of the capital markets and the first fundamental model of asset pricing as a whole. Several popular extensions for the model will be also introduced.
- Arbitrage Pricing Theory (*APT*), which is the multifactor version of the *CAPM* and the second fundamental pricing model, including several extensions.

1.2.1. Expected utility theory of von Neuman and Morgenstern (1944)

Financial decision making under uncertainty is a regular activity of every individual. People cannot predict the future but also cannot avoid facing the future decisions. It is more likely that people build their financial strategies according to some possible expected outcomes. Figure 4 shows the theoretical background for decision making process which leads to the expected utility theory of von Neuman and Morgenstern — a basis for modern capital asset pricing models.

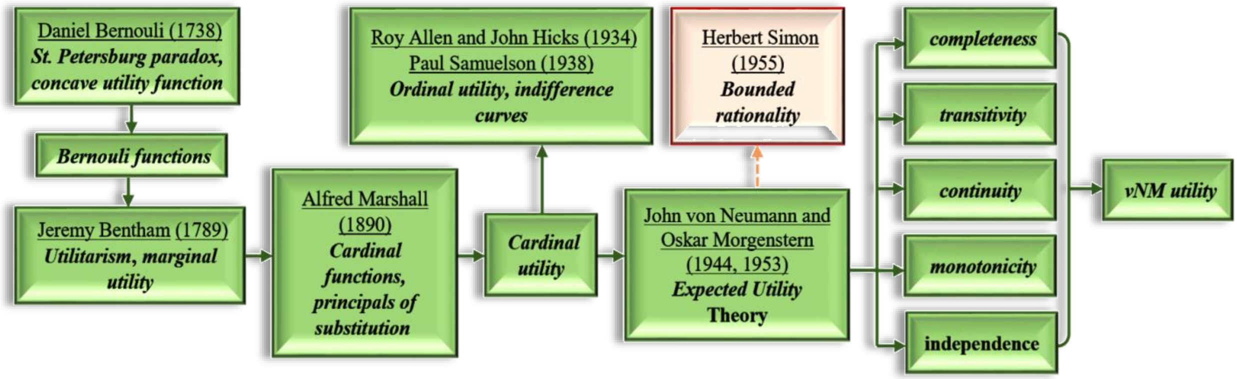


Figure 4. Background for utility theory of von Neuman and Morgenstern
Source: Own work

The first definition for decision making system under uncertainty was made by Daniel Bernoulli (1738). He tries to resolve the *St. Petersburg paradox* for infinite expected utility that was introduced by his cousin, Nicolas Bernoulli in 1713. Particular, he uses a coin toss game to demonstrate the limitation of expected value as a normative decision rule. Bernoulli's analysis of the dynamics of the *St. Petersburg paradox* leads him to appreciate that the subjective value, or utility, that an outcome has is not always directly related to the absolute amount of the payoff or its expected value. Out of his analysis, he proposes a **utility function** to explain a choice behavior of the individuals.

Bernoulli (1738) proposes and applies a concave form for a utility function and uses a logarithmic function of $U(x) = ln(x)$ to represent it. He analyzes such function in order to measure the risks and he realizes that an individual care about the expected utility of possible outcomes rather than about the outcomes themselves. The concave function form allowed him to represent 2 important parameters:

(1) the risk aversion of a person — he makes effort to avoid the risks, when for every agent, it is possible to apply different utility function with different degree of own risk aversion;

(2) the risk premium — dissonance between an individual's utility and taking a part in a gambling.

Indirectly, he means that a decision should be made on a basis of a marginal utility of money, i.e. subjective money value may vary from person to person. In addition, he makes a connection between expected value to its probability, where the risk premium should be higher for events with low probability and vice versa.

In 1789, Bentham publishes a book devoted to the utility issue. There he uses an increasing function to describe the greater happiness or utility from enjoying consuming greater quantities of a divisible good and he argues that the function should be concave due to diminishing marginal utility. Later, it was found that any function that is affine transformation of a given utility function would keep the same characteristics as the original given function. In other words, linear transformation would not harm the initial function. All the family of functions that hold such transformation is called *cardinal functions* and all concave functions that are similar to logarithmic functions are called *Bernoulli functions*.

Cardinal utility approach was proposed by Marshall (1890). He uses it to explain demand curves and the principles of substitution. Marshall (1890) assumes that the utility of each commodity is measurable and the most convenient measure is money. Hence, marginal utility of money should be constant. He also argues that if the stock of commodities increases with the consumption, each additional stock or unit of the commodity gives him less and less satisfaction. It means that utility increases at a decreasing rate. Additionally, he assumes that utilities of goods are independent, where utility obtained from one commodity is not dependent on utility obtained from another commodity, i.e. it is not affected by the consumption of other commodities.

In early 20th century the economists such as Allen and Hicks (1934) and Samuelson (1938) successfully campaigned against notions of cardinal utility, mainly on the grounds that the postulated functions lacked measurability, parsimony, and generality compared to ordinal measures of utility.

Contrary to the notion of cardinal utility formulated by neoclassical economists who hold that utility is measurable and can be expressed numerically or cardinally, ordinalists state that it is not possible for consumers to express the satisfaction derived from a commodity in absolute or numerical terms. Ordinal measurement of utility is qualitative but not quantitative, where the preference is made by ranking the goods. Allen and Hicks (1934) argue that cardinal utility is less realistic because of its inability of quantitative measurement.

Cardinal approach supposes that if a preference can be expressed by a utility function the optimal choice should be based on utility marginal measurement. Contrary, ordinal approach supposes that a choice is a result of comparing between different *indifference curves*. *Indifference curve* is defined as a set of points on the graph, where each is representing a different combination of two substitute goods, which yield the same utility and give the same level of satisfaction to a consumer. Such curves are orderly arranged in a so–called *indifference map* where none of the curves crosses the other. Two conditions are necessary for existence of ordinal utility function: those are ***completeness*** and ***transitivity***. The ordinal utility concept plays a significant role in consumer behavior analysis. Modern economists also believe that the concept of ordinal utility meets the theoretical requirements of consumer behavior analysis even when no cardinal measure of utility is available.

At the middle of the 20th century, just as the ordinalists' victory seemed complete, a small group of theorists including von Neumann and Morgenstern[1] (1944, 1953) build a new foundation for the cardinal utility. Von Neumann and Morgenstern turn the Bernoulli's model assumptions upside down and use preferences to derive utility.

They prove that if a decision maker's risky choices satisfy a short list of 5 plausible consistency axioms, then there exists a particular utility *Bernoulli function* whose expectation maximize those choices. Also, they conduct that only if an individual is rational, his **utility function will hold the necessary axioms**. The axioms were introduced in their publication of

[1] Von Neumann, J., & Morgenstern, O. (1944, 1953). *Theory of Games and Economic Behavior* (2nd ed.). Princeton, US: Princeton University Press.

1947 and as von Neumann and Morgenstern believed, they should be simple, clear and intuitively understandable. The axioms are:

- ***Completeness***: This axiom ensures that for every possible choice, an individual is able to choose between them. A rational individual has an ability to rank all the choices and he will prefer the one that gives him maximum utility. For every new choice, an agent may compare it with the existed and still be able to make a choice. The formal definition is given as follows: For every two lotteries, L_1 and L_2, one should be hold, or $L_1 \prec L_2$ or $L_1 \succ L_2$ or $L_1 \sim L_2$, which is either $L_1$is preferred, L_2 is preferred, or the individual is indifferent between $L_1$and L_2.
- ***Transitivity***: This axiom ensures a choice consistency that is correct for every rational individual. Existing preference cannot be changed and for all possible choices, the most preferred one over others is still be preferred all the time. The formal definition is given as follows: For every three given lotteries, when $L_1 \precsim L_2$ and $L_2 \precsim L_3$ it has to hold that $L_1 \precsim L_3$.
- ***Continuity***: Such axiom ensures that for any gamble, there exists some probability such that an individual is indifferent between the best and the worst outcome. Mathematically, this axiom states that the upper and lower contour sets of a preference relation over lotteries are closed. This axiom is actually a particular case of the *Archimedean property* that says that any separation in preference can be maintained under a sufficiently small deviation in probabilities. The formal definition is given as follows: For every three given lotteries, when $L_1 \precsim L_2 \precsim L_3$ there is a probability of $p \in [0,1]$ such that $pL_1 + (1-p)L_3 \sim L_2$.
- ***Monotonicity***: This axiom ensures that a gamble which assigns a higher probability to a preferred outcome will be preferred to one which assigns a lower probability to a preferred outcome, as long as the other outcomes in the gambles remain unchanged.
- ***Independence of irrelevant alternatives or substitution***: Such axiom ensures that given a preference of one lottery to another, adding the same lottery to the previous should not change the existing preference. Rational individual should concentrate only on those parts that are needed to be compared and to make his choice only with relevant parts. This axiom allows to reduce compound lotteries to simple lotteries. The formal definition is given as follows: For every two given lotteries, when $L_1 \prec L_2$ then for any L_3 and $p \in [0,1]$ it will hold that $pL_1 + (1-p)L_3 \prec pL_2 + (1-p)L_3$.

After defining the axioms, it is possible to introduce the idea of *expected utility function* that also holds the linear transformation. Von Neumann and Morgenstern (1944, 1953) define that if the axioms hold, then it is possible to adjust an expected utility function to a rational individual, which is linear in its probabilities, called *Von Neumann–Morgenstern (vNM)* function and given as follows: A utility function $U: P \to R$ has an expected utility form (the *vNM*) if there are numbers $(u_1, \dots \dots, u_n)$ for each of the N outcomes $(x_1, \dots \dots, x_n)$ such that for every $p \in P$, $U(p) = \sum_{i=1}^{n} p_i u_i$. This implies that:

$$E\big(U(x)\big) = p_1 U(x_1) + \cdots + \ p_n U(x_n). \tag{1.1}$$

If individual's preferences can be represented by expected utility function then the linearity of expected utility means that his indifference curves must be parallel straight lines. The same linearity property also implies that the indifference curves must be parallel and vice versa. So that given the other axioms, the independence axiom is also equivalent to having indifference curves that are parallel straight lines and hence equivalent to having preferences that are representable by a *vNM* expected utility function.

The *vNM Expected Utility Theory* (*EUT*) was applied to normative economies, especially in the game theory, but it has a number of limitations. Assuming individuals' rationality in theory, however in practice, the individuals may do not always behave rationally in the sense of *vNM*.

In 1953, Allais designs a choice problem known as *Allais paradox* to show an inconsistency of actual observed choices with the predictions of the *EUT*. He presents his paradox as a counterexample to the postulate that choices are independent of irrelevant alternatives, i.e. *substitution* axiom, which is the most prominent example for behavioral inconsistencies related to the *vNM* axiomatic model of a choice under uncertainty. The paradox shows that an individual, being rational, prefers not to achieve the maximum expected utility but to achieve absolute reliability.

The empirical evidence on the individual's choice shows that the individuals systematically violate the *EUT*. To response to these findings, several modifications were introduced, mostly by weakening the *vNM* axioms. They include *weighted–utility theory* by Chew (1983); *implicit expected utility* by Dekel (1986) and Chew (1989); *regret theory* of Bell (1982) as well as *rank–dependent utility* theories by Quiggin (1982), Segal (1987, 1989) and Yaari (1987). All of those theories are normative and only the *Prospect Theory* has non–normative basis in attempt to capture individual's attitudes to risky gambles as parsimoniously as possible.

1.2.2. Portfolio theory by Markowitz (1952)

The idea of risk diversification is very old. The bible and the Talmud contain advises, how to divide the investment to avoid the risks. Individuals intuitively understand that holding all their savings in one form of investment could be dangerous, though they do not know what disaster may happen. However, advice of a strategy for risk diversification may be good but does not mean it is efficient. Choosing a portfolio of assets in the stock market is not so trivial. The question of portfolio choice efficiency was very crucial for **Markowitz (1952)**[2]. The work of Markowitz (1952) really has changed a vision of what the stock market was, in the sense of optimal risk adjustment. It was not a guess concept any more, but a true financial market analysis. Today, the method of Markowitz (1952) is known as the ***Modern Portfolio Theory* (*MPT*)** and is recognized as mean–variance analysis.

A process of choosing an optimal portfolio contains 2 stages:

– The first stage starts with an observation as well as experience and ends with beliefs about the future performance of available assets.

– The second stage starts with the relevant beliefs about the future performance and ends with the choice of a portfolio[3].

It is true that an investor attempts to maximize discounted value of future returns. Since the future is uncertain, those values should be replaced with expected returns[4]. Due to price fluctuations, the returns, even expected, may vary. The higher the magnitude, the greater the risk that the desirable expected return will be missed. For this reason, Markowitz (1952) measures the risk as a variance of deviation from some expected average which is expressed in terms of standard deviation[5].

Given N securities, the concept of expected return for any portfolio is expressed as follows:

$$R = \sum_{t=1}^{\infty} \sum_{i=1}^{N} r_{it} d_{it} X_i = \sum_{i=1}^{N} X_i \left(\sum_{t=1}^{\infty} r_{it} d_{it} \right) \to R = \sum X_i R_i,$$

where:

---

[2] Some studies were done before Markowitz (1952) in this direction. For example, Hicks (1935) involves risk measure in his analysis; Marschak (1938), the Markowitz' supervisor, used the means and covariance between goods as utility approximation; Williams (1938), Cowles (1939) and Leavens (1945) who illustrated the benefits of diversification based on the assumption that risks are independent.

[3] Markowitz, H.M. (Ed.). (2008). *Harry Markowitz: Selected works*. World Scientific — Nobel Laureate Series, 1: World Scientific Publishing Co. Pte. https://doi.org/10.1142/6967.

[4] This is the original idea of Williams (1938).

[5] The first who suggested to measure economic risk in the terms of variance was Fisher (1906).

r_{it} – the expected return rate at time t of security i;

d_{it} – the discount rate;

X_i – the amount of money spent in i;

R_i – the discounted return and independent of X_i.

Therefore, R is the weighted average of R_i with non–negative weights of X_i, hence R should be maximized. Returns are random variables, i.e. its value is generated by a chance. The proportions of the assets in a portfolio are decided and fixed by an investor. In this case, two variables are to be determined: the expected return which is given by:

$$E(R) = \sum_{i=1}^{n} X_i \mu_i$$

and its variance, given by:

$$V(R) = \sum_{i=1}^{n} \sum_{j=i}^{n} X_i X_j \sigma_{ij},$$

where:

$$\sum_{i=i}^{n} X_i = 1.$$

Another important decision factor for better assets combining is the correlation between the chosen assets which is expressed by the covariance $\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$. When the covariance coefficient ρ is equal to 1, perfect correlation occurs and this is the worst scenario for an investor. The covariance measures diversification when the best diversification is obtained with uncorrelated assets, i.e. the covariance coefficient ρ is equal to (–1). Before this method, *simple* or *naive diversification* took place. The investors intuitively understood the advantages of such action but no measurement or criterion existed.

Markowitz (1952) makes three main assumptions:

(A1) – selling in short position is impossible. This ensures that all weights of chosen assets in a portfolio will be non–negative, hence the sum of all the weights equals to 1;

(A2) – all the individuals are risk averted. This is behavioral definition of individual's choice (with underlying rationality assumption);

(A3) – an individual should minimize a volatility which is measured in the sense of variance (or standard deviation) and represents risk; or maximize the expected return in a given level of variance.

According to the assumptions, Markowitz (1952) constructs an ellipsoidal line that contains all mean–variance efficient portfolios set, called the *efficient frontier* (though Markowitz himself did not call it that way), which is given by minimizing the following equation:

$$X^T \Sigma X - qR^T X, \tag{1.2}$$

where:

X – a vector of portfolio weights with the sum of all weights is equal to 1;

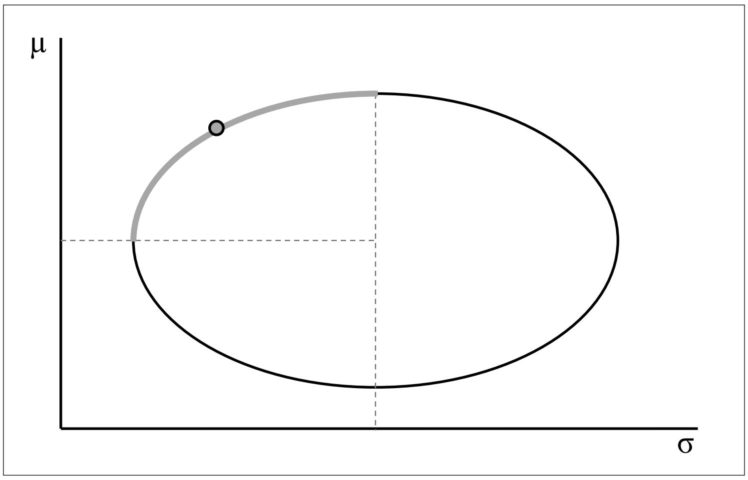
Σ – the covariance matrix between the returns of the assets in the portfolio;

q – a non–negative risk tolerance factor;

$R^T \omega$ – the expected return on the portfolio.

All over the *efficient frontier* it is possible to obtain a higher *ROR*, in a given level of standard deviation. As Markowitz (1952) suggests, an investor should make his portfolio choosing with regard to the *efficient frontier*.

However, none of the whole ellipsoidal line is appropriate for the portfolio choosing. Among all possible mean–variance efficient options only the optimal portfolios are acceptable which are located on the grey part of the ellipsoidal line as shown in the Graph 1.



Graph 1. The *efficient frontier* and the optimal portfolios set
Source: Own work

For Markowitz the efficiency was not the only problem. Additionally, he was looking for the optimal investment choice. In order to determine it, he resolves next optimization problem:

$min\{\sigma^2 - A\mu\}$, where A is a degree of personal and unique risk aversion. When *risk–free* lending and borrowing is possible with zero variance and return *r*, then the combination of a portfolio with riskless asset may improve the diversification. In this case, the expected return may be written as:

$$E(R) = r + \frac{\mu_p - r}{\sigma_p}\sigma. \tag{1.3}$$

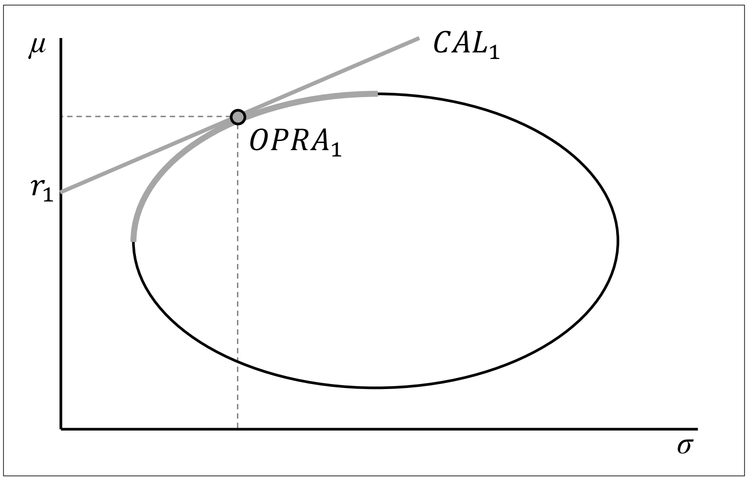
Here, maximizing the expected return for a fixed level of risk is equal to maximizing its slope and the optimization problem accepts the next form:

$$ma\,x\left\{\frac{\mu_p - r}{\sigma_p}\right\},$$

$$Subject\ to\ V(R) = \sum_{i=1}^{n}\sum_{j=i}^{n} X_i X_j \sigma_{ij},$$

$$\sum_{i=i}^{n} X_i = 1.$$

A portfolio giving μ_p and σ_p that maximizes the slope is called *Optimal Portfolio of Risky Assets* (*OPRA*). Giving a *risk–free* asset, *OPRA* is a portfolio on the *efficient frontier* that is connected with a line to a *risk–free* asset, as shown in the Graph 2. Such line reflects all optimal weighted portfolios within a given risk level and a *risk–free* asset. This line is called *Capital Asset Location* (*CAL*) and Markowitz's suggestion is choosing a portfolio within the *CAL*.



Graph 2. The *Capital Asset Location (CAL)* line and the *Optimal Portfolio of Risky Assets* (*OPRA*)
Source: Own work

Despite a great contribution of the *MPT* to the theoretical and mathematical understanding of a stock market, a number of criticisms is addressed to Markowitz (1952). The first criticism is concentrated on the lack of parameters to determine an optimal portfolio. Many necessary macroeconomic factors, such as inflation, income, etc., are not involved in the process and nor the fundamental analysis as Graham and Dodd (1934) suggested.

Additional strong criticism refers to a doubt whether a volatility is a good measure of a risk. Standard deviation, as a symmetric estimator, measures the upside movements as equally bad as the downside movements. Obviously, for an investor upside movement is desirable but the downside movement should represent a real risk exposure. Sometimes, downside movement could be viewed differ if before, an investor won upside movement. ***Post–Modern Portfolio Theory*** **(*PMPT*)** uses downside risk of returns which is based on the Markowitz' (1959) semi–variance instead of the mean–variance to resolve the problem (see Estrada (2002, 2007)). Finally, speculative stocks which are extremely volatile do not fit into this format as they certainly do not give superior returns, as a diversified group or otherwise.

Another problem refers to the assumption of returns distribution that is supposed to be normal and symmetric. In the reality, the stock returns not always follow a normal distribution. There is a considerable evidence of skewness, fat tails, kurtosis etc. The volatility cannot reasonably be predicted from a normal distribution. Mandelbrot (1963) and Fama (1965) are the first who emphasize that the stock returns are not following the Gaussian distribution. Further early studies, including Officer (1972), Clark (1973), McCulloch (1985) and Bollerslev (1987), demonstrate that the assumption of returns normal distribution may be insufficient.

1.2.3. CAPM

The *CAPM* is the central fundamental model, which has conquered the academia for decades. Despite the fact that today the investors relay less on the model, it pushed the development of other important normative models (see Figure 5).

The ***Capital Asset Pricing Model*** **(*CAPM*)**, introduced separately by **Treynor (1961, 1962)**, **Sharpe (1964)**, **Lintner (1965)** and **Mossin (1966)**, is the logical evolution of the Markowitz's (1952) *MPT*. Hence, a strong connection and similarity between the *CAPM* and the *MPT* are present. The goal of the model is to determine a required rate of return to justify adding an asset to an already well–diversified portfolio. Another proposal of the model is a determination of compensation for bearing extra risks above the *risk–free* rate contrary to the

MPT that suggests only optimal diversification, when the higher the risk is accepted, the higher the expected compensation above the *risk–free* rate.

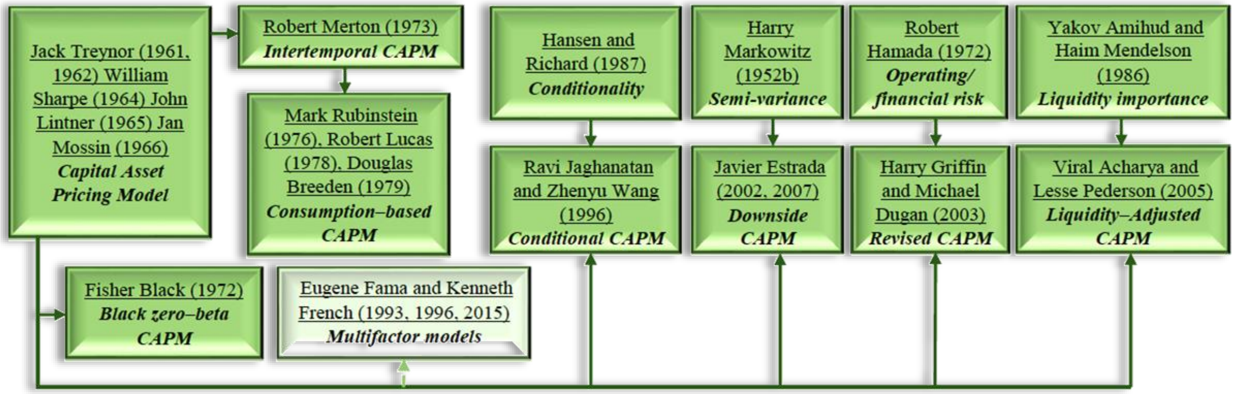


Figure 5. The CAPM and its extensions
Source: Own work

The theoretical framework of the model starts with a set of assumption, which are extremely important to understand its inner logic:

(A1) – all investors have the same expectations and they see the same probability distribution of returns, when their expectations are all rational;

(A2) – investors are interested to maximize expected return within a given level of volatility or to minimize volatility for a given level of expected return;

(A3) – there is non–limited possibility to lend and to borrow at the *risk–free* rate;

(A4) – all investors have the same 1–period investing horizon as day, month and so on;

(A5) – all assets are infinitely divisible;

(A6) – there are no taxes or any costs;

(A7) – interest rates are unchanged and the inflation has no influence or fully anticipated;

(A8) – the *EMH* is valid and the markets are in constant equilibrium.

Several assumptions, like lending and borrowing at the *risk–free* rate or absence of taxes and costs, are out of economic logic. Of course the founders realized that. For them, it was not a problem because their goal was to create a sufficient framework based on mean–variance analysis and to define the equilibrium in capital markets. Further, it will be possible to create more sufficient model by relaxing the unrealistic basic assumptions.

Markowitz (1952) suggests to choose a portfolio within a *CAL*, which is the line that contains all possible combinations between a giving *risk–free* asset and responding tangency portfolio on the *efficient frontier*. The *CAPM* defines the tangency portfolio as a theoretical *market portfolio* (*Mkt*) that contains of all possible assets in the market, including human power, houses, etc. Since the fact that the *market portfolio* is a point on the *efficient frontier*, it is supposed to be at maximum level of diversification, hence optimal. This time, the same *CAL* becomes the *Capital Market Line* (*CML*) and contains of all optimal choices where unsystematic risk is totally eliminated.

Next step is looking closer to a portfolio content in the context of individual assets. A portfolio, as a part of the *CML*, has only **systematic risk** that cannot be diversified away. This systematic risk is represented by $\beta_{Pf} * \sigma_{Mkt}$ where the *beta* *(*β_{Pf}*)* of a portfolio is the covariance between the portfolio return and the market return divided by the variance of the market return. *Beta* is a measurement of a correlation between the price fluctuations of a single security and the average price fluctuations of overall security market, which is given by next equation:

$$\beta_i = \frac{Cov(R_i; R_m)}{Var(R_m)}, \tag{1.4}$$

where:

R_i – the return on a security i;

R_m – the market return.

In addition, the *beta* measures the asset's statistical variance which cannot be spread by diversification provided by the portfolio of risky assets. At the same time, *beta* is also used to reflect the elasticity between the return rate of individual security and that of the whole market. There are next possibilities for the *beta*:

$\beta_i > 1$ – the fluctuation range of asset's return rate is larger than the market average and its expected payoff is over the market average level, it is called ***aggressive asset***;

$\beta_i = 1$ – the fluctuation range of an asset's return rate equal to the market average and its expected return is the same as the market average level;

$0 < \beta_i < 1$ – the fluctuation range of an asset's return rate is less than the market average, and its expected payoff is below market average level, it is called

defensive asset;

$\beta_i = 0$ – the return rate is the same as of the *risk–free* asset;

$\beta_i < 0$ – the portfolio and market returns move to the opposite directions.

In the equilibrium, all securities and all portfolios are on the same ***Security Market Line* (*SML*)**, which is defined mathematically by the next equation:

$$SML\text{: } E(R_i) = R_f + \beta_i\big(E(R_m) - R_f\big)\,, \tag{1.5}$$

where:

$E(R_i)$ – the expected return on a security;

R_f – the *risk–free* rate;

$E(R_m)$ – the market rate of return;

β_i – the non–diversifiable or systematic risk.

Additionally, the *SML* indicates how an investor should be compensated for bearing a unit of risk in the terms of *beta*, but not in the terms of volatility. The difference $E(R_{Mkt}) - r_f$ is called *market risk premium*.

While portfolio volatility is absolute measurement that responds to its returns, *beta* is a relative measurement which measures the portfolio movement relatively to the market. All the investors see the same *SML* of optimal portfolios and make their decision with the same criterion. Individual optimal portfolio construction, adjusted to the individual risk profile, is possible to create with ***Tobin Separation***, which is the process of finding an optimal mix of market assets that do not vary with risk tolerance with appropriate amount of cash combining.

The *CAPM* makes connection of a portfolio return to the market return through its correlation with the market. All portfolios suggested by the model are optimal, but investors and as well as fund managers may have a mistaken decision and not to apply such portfolio. In this case, the investment performance can be measured as an aberration from the *SML*. At least three ratios are often in use to achieve this goal. Sharpe (1966) develops a *reward–to–variability* ratio that is recognized as ***Sharpe ratio*** in the present days and is given by the following equation:

$$S_a = \frac{R_a - R_f}{\sigma_a}, \tag{1.6}$$

where:

R_a – the asset return;

R_f – the *risk–free* asset;

σ_a – the standard deviation of the asset.

The ratio shows what excess return was achieved per unit of risk. Similar approach was introduced by Roy (1952) where he uses *Minimum Acceptable Return* (*MAR*) instead of the *risk–free* asset. The Roy's *MAR* is a "disaster level" that an investor should decide about it and maximize it, when the *Sharpe ratio* is all given by the market. The *Sharpe ratio* was corrected by its author in 1994 in so–called ***information ratio*** that is a measure of the risk–adjusted return of a financial security and estimates ex–post value added and relates this to ex–ante opportunity available in the future. It is given by the next equation:

$$S_i = \frac{E(R_a - R_b)}{\sigma_a}, \tag{1.7}$$

where:

R_b – the benchmark asset return; any stock index may play a role of the benchmark asset.

Another popular ratio is the ***Treynor ratio*** (sometimes *Treynor measure*), which is developed by Treynor in 1966. This ratio shows what an excessive return over the *risk–free* asset was achieved per a unit of systematic risk when high *Treynor ratio* indicates better portfolio performance. It is given by the following equation:

$$T = \frac{R_i - R_f}{\beta_i}, \tag{1.8}$$

where:

R_i – the asset return;

β_i – the portfolio *beta*.

Jensen's alpha is the ratio that measures abnormal returns of portfolio over the returns that are suggested by the *CAPM*. It was introduced by **Jensen (1968)** as follows:

$$\alpha = (R_i - R_f) - \beta_i(R_M - R_f). \tag{1.9}$$

The *alpha* is a statistic that is commonly used in empirical finance to assess the marginal return associated with unit exposure to a given strategy and obtained by a regression.

Despite some successful empirical hits at the beginning, the *CAPM* has suffered the most from **critique**. Unrealistic assumptions naturally make the model unrealistic too. The supporters of the model ascertain that the assumptions are not the crucial part of the *CAPM*. The central part was to show how risk and returns are connected to each other through the market correlation, when the risk is measured in the terms of market relative volatility. However, the question of how the volatility could be a good measure for risk stays opened.

Another set of criticisms concentrates on the *beta* coefficient. *Market portfolio* is a theoretical and hypothetical object. Therefore, it is impossible to measure such abstract object, hence the correlation coefficient cannot exist. *Beta* also has estimating problems: it is estimated under rational expectations and there is no logical justification that an individual is rational. It is estimated by a liner regression due to a normal distribution of returns that is not necessarily true in reality. Here, the issue of problematic observability of a *market portfolio* which harms the estimation but suggests using a proxy that does not match the reality.

For the last 50 years academics and practitioners have been debating the merits of the *CAPM*, focusing on whether the *beta* is an appropriate measure of the systematic risk. Researchers, in general, always find a weak, but positive relationship between *beta* and returns over the sample period, as shown in Fama and French (2004). For such weakness, they claim that the results are inconsistent with the positive linear relationship between *beta* and returns as prescribed by the *CAPM* and the validity of the model is in question. With it, there is an empirical evidence (Fama and Macbeth, 1973) that *beta* cannot be equal to zero.

Due to the unrealistic and oversimplified assumptions of the traditional *CAPM* some extensions appeared in an attempt to fix the models' bungling. The most popular are:

- Black zero–beta CAPM by Black (1972),
- Inter–temporal CAPM (ICAPM) by Merton (1973),
- Consumption CAPM (CCAPM) by Rubinstein (1976), Lucas (1978) and Breeden (1979).

First extension is known as ***Black zero–beta CAPM*** of **Black (1972)**. He assumes that instead of the *risk–free* asset there is some portfolio S, which is uncorrelated with the market and hence, has zero *beta*. Among several *zero–beta* portfolios only one should own minimum variance and to be represented by S. His model is given by the next equation:

$$E(R_p) = E(R_s) - \beta_p[E(R_M) - E(R_s)], \tag{1.10}$$

where:

$E(R_p)$ – the portfolio return;

$E(R_s)$ – the return rate of the portfolio with zero beta;

$E(R_M)$ – the market return.

Merton (1973) introduces another version of the model called ***Inter–temporal Capital Asset Pricing Model*** (***ICAPM***) which is the dynamic version of the *CAPM*. He assumes that investors concern the uncertainty of the asset's price future other risks, which may have an impact on their future consumption or investment. The model contains all kinds of non–market risks and suggests on optimal consumption combination by the following equation:

$$E(R_p) = R_f + \beta_{p,m}[E(R_M) - R_f] + \beta_{p,f_1}[E(R_{f_1}) - R_f] + \ldots + \beta_{p,f_n}[E(R_{f_n}) - R_f], \quad (1.11)$$

where:

R_f – the *risk–free* rate with (f_1, f_2…f_n) as representation of extra market risks;

n – the number of elements or extra market risk sources;

$\beta_{p,m}$ – the sensitivity of the portfolio to market;

β_{p,f_n} – the sensitivity of the portfolio to element n;

$E(R_{f_n})$ – the expected return of risk n;

$E(R_p)$ – the expected return of the portfolio.

Another extension of the *CAPM* was introduced by **Rubinstein (1976)**, **Lucas (1978)** and **Breeden (1979)** by proposing that investors maximize their consumption utility $U(C_t, C_{t+1}) = u(C_t) + \beta E_t[u(C_{t+1})]$ during whole lifetime. The model ***Consumption–based Capital Asset Pricing Model*** (***CCAPM***) is constructed based on the traditional *CAPM*, as well as on the *ICAPM*. The *CCAPM* suggests an existence of a positive linear relation between anticipated returns on securities and the average growth rate of consumption, where both should move in the same direction. Breeden's (1979) model demonstrates dependence of anticipated rate of return upon its covariance with the marginal utility of consumption which is given as:

$$E(R_i) = R_f + \frac{\beta_{i,lnC}}{\beta_{m,lnC}}[E(R_m) - E(R_f)]. \quad (1.12)$$

1.2.4. APT

One of the prominent problems of the *MPT* and subsequently of the *CAPM* is that the models make a connection between only two parameters and totally avoid other important economic factors. Assumption that an agent is interesting only in a minimal volatility can be insufficient. All agents act under the global economic environment and so does the market. This means that some fluctuations in the stock prices can take place controversially to expected prediction of the *CAPM*. The central idea behind the *APT* is that some assets are priced relatively to other assets. According to Ross (1976), in an economy with a large number of available assets, a linear factor model of asset returns implies that particular risk is diversifiable and that the equilibrium prices of securities will be more or less linear in their factor exposures.

Ross (1976) suggests that the correlation of the returns should be done due to the different economic circumstances. Each economic circumstance has its own correlation. This approach was introduced by **Ross (1976)** as the ***Arbitrage Pricing Theory*** **(*APT*)** which is a multifactor model based on the principle of one price with no arbitrage. In this sense, the *CAPM* and especially the *ICAPM* can be seen as special cases of the *APT*. Theoretically, the number of the factors is infinite, but Ross (1976) argues that in the long run there is only limited number of factors which are really influential and associated with expected returns. Ross (1976) calls those long–run influential factors as *underlying economic forces*. The main goal is to determine the *underlying economic forces* and to figure out the expected returns through its correlation with the forces.

The intuition of the *APT* is based on Arrow–Debreu (1954) security pricing. *A* set of *K* fundamental assets should cover all possible states of nature. When no arbitrage is allowed, then the current price of each asset is the weighted average of the current prices of the fundamental assets. Such intuition can be seen in terms of returns and expected returns rather than in terms of prices. If the unanticipated part of each asset's return is a linear combination of the unanticipated parts of the returns on the *K* fundamental securities, then the expected return of each asset is the same linear combination of the expected returns on the *K* fundamental assets.

The *APT* is derived from a statistical model whereas the *CAPM* is an equilibrium asset pricing model. According to the model, there are multiple factors and for each factor different intensity due to its correlation with an asset. Such correlation may be represented by a specific coefficient as β in the sense of the *CAPM*. If so, the mathematical representation for the risky asset return is given by the next equation:

$$R_j = E_j + b_{j1}f_1 + b_{j2}f_2 + \cdots + b_{jn}f_n + e_j, \quad (1.13)$$

where:

R_j – the risky asset's return;

E_j – the expected return on the asset;

b_{j1} – the sensitivity to a change in a systematic factor;

f – the actual return on the systematic factor;

e_j – the return on unsystematic, idiosyncratic factors that are assumed to be uncorrelated across assets and uncorrelated with the systematic factors.

Ross (1976) proves that expected return on any asset is directly related to that asset's sensitivity to unanticipated movements in major economic factors, so the total expected return (E_j) on the portfolio, may be computed as:

$$E_j = r + b_{j1}(Ef_1 - r) + b_{j2}(Ef_2 - r) + \cdots + b_{jn}(Ef_n - r). \quad (1.14)$$

The equation simply states the next relationship: the expected return on any asset, E_j exceeds the riskless return r by an amount equal to the sum of the products of the market prices of risk, $E_jf_n - r$, and the sensitivities (b_j) of the asset to each of the respective factors.

The model has fewer assumptions than any *CAPM* version, hence it is more flexible:

(A1) – there are no taxes or any costs. (This assumption is the same as of the *CAPM*);

(A2) – all assets have finite expected values and variances;

(A3) – some individuals can form well diversified portfolios.

Roll and Ross (1995) point out 4 most important ***underlying economic forces*** that are the primary influences on the stock market:

(F1) – the unanticipated inflation;

(F2) – the changes in the expected level of industrial production;

(F3) – the unanticipated shifts in risk premiums;

(F4) – the unanticipated movements in the slope of the term structure of interest rates.

Roll and Ross (1995) argue that these variables are indeed systematic. They point that the factors have a strong connection to the traditional *Discount Cash Flow* (*DCF*) valuation formula. Forces (F1) and (F2) are associated with the expected cash flow in the formula where the expected level of industrial production is a proxy for the real value of future cash flow and the inflation enters since the assets are not *neutral*. Forces (F3) and (F4) are associated with the risk–adjusted discount rate. The risk premium is the ratio of an investor's attitude toward risk–bearing and individual perception of general level of uncertainty. The term structure of interest rates enters since most assets have multiple year cash flows and, for reasons relating to risk and time preferences, the discount rate that applies to distant flows is not the same as the rate that applies to flows in the near future. In other words, the influential underlying economic forces complete risk adjusted *Net Present Value* (*NPV*) of an investment in macroeconomic terms.

Despite the suggestion of 4 main underlying forces, however, there is no formal theoretical guidance in choosing the appropriate group of economic factors. Therefore, in practice, it is left to empirical researchers to make up their own personalized *APT* model and test it according to their individual intuition about the underlying forces involved in the model. This led to a great disagreement between the researchers about better explanatory variables.

Roll and Ross (1980) determine the differences between the *CAPM* and the *APT*. The *APT* is based on a linear return generating process as a first principle and there is no requirement for utility assumptions and limitation of time. There is no importance to some specific portfolio in the *APT* contrary to the *CAPM*. The model avoids the problematic *market portfolio* because there is no requirement that the *market portfolio* should be mean–variance efficient. The *APT* demonstrates that since any market equilibrium must be consistent with no arbitrage profits hence, the equilibrium will be characterized by a linear relationship between each asset's expected return and its return's response amplitudes, or loadings, on the common factor.

The *APT* as the *CAPM* turned to a fundamental model though has not won a lot of recognition in academia as the *CAPM* did. The most problematic part is that the factors are not truly defined, it is better to guess what single factor has an influence on a particular asset.

This problem was partially solved by **Fama** and **French,** who in **1993** introduced the **three–factor model** for asset pricing. Their model, more econometric or statistical rather than a theoretical framework, was created consequently to the results of their research. The three–factor model is created by adding two more variables to already existed *CAPM beta* coefficient and by this generalizes the *CAPM* where the *APT* is a private case.

The Fama–French model combines the idea of both fundamental models — *CAPM* with *APT*. That is because Fama and French believe that the inner logic and concept of the mean–

variance analysis are generally right, but the models miss the reality and needed to be improved. This point was highlighted in 2004 when they finally conducted that the empirical record of the *CAPM* is poor enough to invalidate the way it is used in applications[6]. Even though today, the *CAPM* is still being the most used model both by the academia and the industry despite its failure.

In the early 80s, so–called anomalies are started to surface, questioning the efficiency of the *beta* in measuring the risk. For example:

- Basu (1977, 1983), who discovered the *value effect* where different financial ratios have higher explanatory power on the prices than the *beta*.
- Statman (1980), Rosenberg, Reid, and Lanstein (1985) and DeBondt and Thaler (1987) reported the relation of positive abnormal returns that are possible to occur to the portfolios of stocks with high book–to–market (*B/P*) values.
- Banz (1981), Reiganum (1981) and Keim (1983) reported the *size effect* where small–capitalization firms earned higher average returns than they should to earn according to the theory.
- Bhandari (1988) through a measure of leverage showed that high debt–equity ratios may generate high returns relatively to their market *betas*.

Fama and French (1992) begin to examine the anomalies and empirically found that there are two of them have the greatest influence on returns and should be necessary included in the calculation of an expected return. The anomalies are the *size effect* and the *book–to–market* ratio. Later, Fama and French (1993, 1996) introduce their three–factor model where the first factor is equal to the standard market *beta* and two additional factors represent the size and the *B/P* influences. Their model is mathematically given by the next equation:

$$r = R_f + \beta_3(K_m - R_f) + b_s \cdot SMB + b_v \cdot HML + \alpha, \tag{1.15}$$

where:

r – the portfolio's expected rate of return;

R_f – the *risk–free* return rate;

K_m – the return of the *market portfolio*;

[6] Fama, E.F., & French, K.R. (2004). The capital asset pricing model: Theory and evidence. *Journal of Economic Perspectives, 18*(3), 25–46.

β_3 – the correlation in the vein of classical β but not equal to it;

SMB – the historic excess returns of small caps over big caps;

HML – the historic excess returns of value stocks over growth stocks;

b_s , b_v – the sensitivity coefficients that can be negative or positive.

Adding those two factors significantly increased the explanatory ability of the standard model. For this reason, Fama and French (1993, 1996) conclude that in the standard model of *CAPM* probably are missing size and the *B/P* factors. Fama and French (1994) extend their conclusions to industries and Fama and French (1998) apply the model to the international markets where they reach the same conclusions. Today, their model is one of the leading normative methods and their results turned to a benchmark for further asset pricing models.

Despite its success, the three–factor model was extended by Carhart (1997) to a four–factor model and by Fama and French (2015) to a five–factor model. Those extensions raised as an answer to more and more anomalies that troubled the original *CAPM*, revealing additional risk factors that potentially omitted in the *CAPM* and still be not captured by the three–factor model. Hence, **Carhart (1997)** suggests to add a momentum factor (*WML* – winners mines losers), making his model written as follows:

$$r = R_f + \beta_3(K_m - R_f) + b_s \cdot SMB + b_v \cdot HML + b_m \cdot WML + \alpha, \tag{1.16}$$

Fama and French (2015) suggest to add to their original three–factor model also *RMW*, which is responsible for the profitability (as robust minus weak) and *CMA*, which is the investment (as conservative investment minus aggressive) where the model is given by:

$$r = R_f + b_i(K_m - R_f) + s_i \cdot SMB + h_i \cdot HML + r_i \cdot RMW + c_i \cdot CMA + \alpha_i, \tag{1.17}$$

In a case where the sensitivity coefficients of all variables are able to explain the variation of expected returns, α_i should be equal to zero for all securities and portfolios i. Notably, these extended version does not include momentum factor as in the model of Carhart (1997).

The standard *CAPM* assumes that all investors are likely to choose their efficient portfolios relatively to the average variance. Such approach is criticized by Bawa and Lindenberg (1977), Kaplanski (2004) or Bornholt (2007). Particular, **Bornholt (2007)** derives a class of average risk measures based on the *APT* where the *CAPM* is a special case. He proves the consistency of such measures with the *EUT* and the hypothesis of risk aversion. In 2007 Bornholt introduces the ***Reward Capital Asset Pricing Model*** (***Reward–BETA***) where the main difference from the fundamental theory is the method of calculation for the *beta*.

Bornholt (2007) argues that a stock return is the sum of two parts where the first part is expected returns and unexpected returns is the second part. By assuming that the *CAPM* is true, he rewrites the *beta* referring to the average risk instead, as follows:

$$\beta r_i = \frac{E(R_i) - R_f}{E(R_m) - R_f}, \tag{1.18}$$

and the model can be represented by the next equation:

$$E(R_i) = R_f + \beta r_i\left[E\left(R_m - R_f\right)\right] + \beta_i[R_M - E(R_m)]. \tag{1.19}$$

In the model, the expected return of the asset i is determined by the ***Reward–BETA***, the *risk–free* rate and the premium for the market risk. The coefficient βr_i correlates with the asset's i return volatility and controls the covariance between the asset's and the market's return, without to affect the expected value. This means that even the traditional *beta* can be used *ex–post* to adjust the data for the model, it is not *ex–ante* relevant to estimate the expected returns. In a case of the *CAPM* dominance, the *Reward–BETA* model can be reduced to the standard version of the *CAPM*.

Using Fama–French methodology of sorted portfolios, Bornholt (2007) concludes that the *Reward–BETA* model has higher predictive power than the standard *CAPM*. Rogers and Securato (2007) replicate the study of Bornholt (2007) and totally confirm his findings. In their next study, Rogers and Securato (2009) add to the *Reward–BETA* model size and book–to–market factors to compare the results with the Fama–French model and conclude superiority of the *Reward–BETA* model over both the *CAPM* and the three–factor models.

1.3. Behavioral theories of capital asset pricing

The behavioral theory, exactly as the traditional theory, starts with the *Theory of Moral Sentiments* (1757) and the *Wealth of Nations* (1776) by Adam Smith. In his work, Smith proposes the definition for the economic behavior, but also touches other aspects (see Figure 6). He talks about the morality of individuals that guides them in making social, economic and even financial decisions. Another improved thinking in that period was highlighting the psychological aspects of utility function. The decision making cannot avoid human emotions. Further, this concept laid in the foundation of the behavioral finance as a whole.

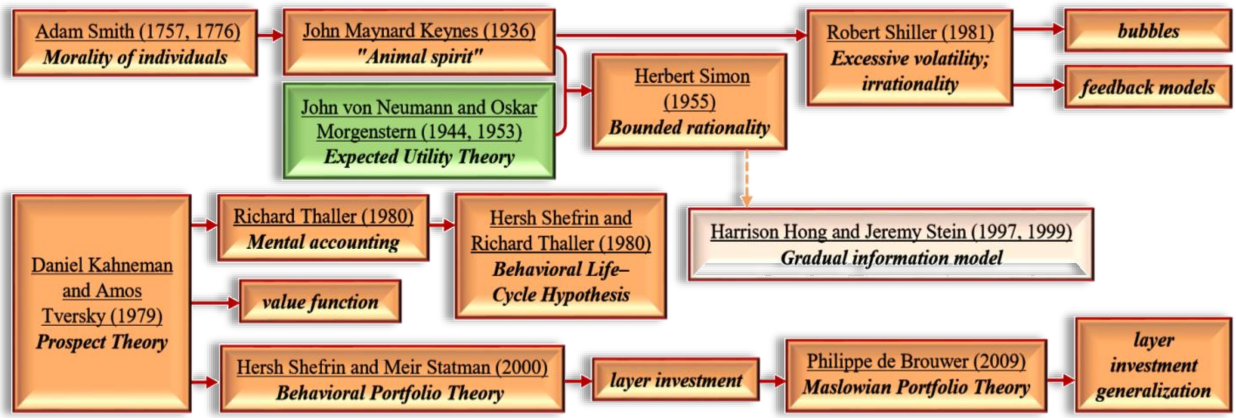


Figure 6. Evolution of theoretical background for behavioral finance
Source: Own work

Keynes (1936) points out the role of sentiment as the "animal spirits" of individuals. He describes by this term the non–rational components of human being that may influence and guide human behavior. Such components can be measured, for example, by a consumer confidence. Keynes (1936) criticizes the concept of *Homo economicus* and argues that human beings cannot be completely informed of every situation in order to maximize their expected utility. For example, Akerlof (1970) through "*The Market for Lemons*" explains a lack of informational availability by asymmetry.

Not so far after introduction of the *EUT* of von Neumann Morgenstern, Simon (1955) calls into a doubt human rationality within the *Homo economicus* concept. He constructs a model, known as *bounded rationality*, where he puts two elements to describe a "real man" thinking:

- the first element is a set of behavior alternatives;
- the second element is a choice that is built only on a segment of available information.

According to the theory, human beings have cognitive limitations of their own minds and the information they have at their disposal. In this aspect, the individuals prefer to maximize their satisfactory rather than their expected utility. The *bounded rationality* is the concept that rose on the idea of an impact of a rational individual on natural limits, like inability to obtain the information at the same time with other individuals or inability to access full information. In this case, an individual’s decision is based only on partial information and he has limited number of choices.

Simon (1955) believes that concept of *limited rational man* is closer to real–life thinking though he did not exclude the rationality totally. *Bounded rationality* is more relaxed version of the standard *EUT* of *vNM* and hence is much more realistic than the traditional concept.

Psychological aspects of human economic decisions are widely investigated by different researches in their works like Maslow (1943), Allais (1953), Simon (1955, 1957), and Samuelson (1963). However, the real breakthrough was made by **Kahneman** and **Tversky** in **1979**. They investigated the field of making decision under uncertainty and shared the results of their work in the form of ***Prospect Theory*** which has become the very foundation of the **behavioral finance**. Kahneman and Tversky (1979) found out that during the real case study the individuals constantly violate the axioms of the *EUT*. Individual choice is based on some inner probability weighted *value function* rather than on the expected utility where the sum of the probability weights can fall less than a unity. The individuals have no ability to determine the probability. They are likely to overweight probability in its low levels but underweight it in its high levels. Kahneman and Tversky (1979) also found that contrary to a utility function, a value function has both positive and negative domains. In its positive domain individuals are likely to be risk averted and in its negative domain they are likely to be risk takers, where a function slope is steeper than in its positive domain.

Since both Kahneman and Tversky were psychologists with no or little training in traditional finance, **Thaler (1980)** makes the passage from the psychological findings to the definition of the behavioral economics. Thaler is best known as the theorist of the behavioral finance. In 1985, based on the *Prospect Theory*, Thaler establishes the concept of *mental* (or psychological) *account*. According to the concept, the individuals divide all portions of possible goods and assets, whatever present or future, into groups and assign different levels of utility to each single group. Those portions are non–transferable.

The central idea of mental accounting is that through the same nominal value, money may have different mental value. Hence, the individuals treat their money differently, depending on its origin and intended use. For example, a lottery big win may be spent for purchase of high–cost goods meanwhile the same goods cannot be purchased with money from a salary. Individuals think that the assets are less interchangeable than they really are by *framing* them as its belonging to current wealth, current income, or future income.

Continuing incorporation of the ideas from the psychologists like Kahneman and Tversky into working theories, **Shefrin and Thaler (1988)** introduce the ***Behavioral Life–Cycle Hypothesis*** **(*BLCH*)**, that can be traced back to Modigliani and Brumberg (1954). The *LCH* is the approach that presumes that individuals base their consumption on a constant percentage of their expected life income, meaning the individuals base their consumption/saving decisions on their lifetime resources rather than on the current income. Contrary, the *BLCH* involves *mental accounting*, *framing* and *self–control* for the same propose.

In 1987, the Roy's *SFPT* receives psychologically explained extension, made by Lopes (1987), named *SP/A Theory* of a choice under uncertainty, where:

S – the *security* — a general concern about avoiding low levels of wealth;

P – the *potential* — reflects the general desire to maximize wealth;

A – the *aspiration* — the desire to reach a specific goal, such as achieving no less than the subsistence level *S*.

Lopes (1987) defines 2 forces that are consistent with a *value function* of Kahneman and Tversky (1979). The concave component of a *value function* is associated with *fear* and pushes an individual to search for *security*, making him risk–averted person while convex component of a *value function* is associated with *hope* that pushes an individual to reach some desirable level of *potential*, making him risk–seeker. Lopes (1987) posits that *fear* causes individual to excessively overweight the probability of the worst outcomes and to underweight those for the best outcomes while *hope* has the inverse effect on individuals.

In her work, she postulates that risky outcomes are evaluated in terms of 2 variables:

- The first variable is $E_h(W)$, the expected value of wealth (W) under the transformed decumulative function $h(D)$.
- The second variable is $D(A)$, that is the probability that the payoff will be A or higher. The same two variables are virtual analogues of the arguments used in the *SFPT*, $E(W)$ and $Pr\{W \leq s\}$.

The *SP/A* framework is actually similar to the *Value at Risk (VaR)* framework. Both theories suggest optimization involving tradeoffs between expected wealth and probabilities of falling below of a given *aspiration* level. The *aspiration* level of the *SP/A* in *VaR* analyses is associated with a poverty level. The main propose is to combine a low probability of falling below a poverty level with the highest possible expected wealth.

One more prominent application of the *mental accounting* was made by **Shefrin and Statman (2000)** in their ***Behavioral Portfolio Theory*** **(*BPT*)**. The *BPT* is the sort of mean–variance analysis that involves Roy's (1952) safety–first criterion and *SP/A* model of Lopes (1987). In 1952 were introduced 3 portfolio theories where the Markowitz's (1952b) *CWT* and the Roy's (1952) *SFPT* are consistent with the **Friedman–Savage (1948) puzzle** and only the

Markowitz's (1952) *MPT* is not. In reply to Markowitz (1952), Shefrin and Statman (2000) introduce a *behavioral efficient frontier* contrary to mean–variance *efficient frontier*, which is similar to those of Markowitz (1952), but with some different aspects. They also prove that the portfolios chosen with their approach are really optimal and efficient.

Shefrin and Statman (2000), introduce the *Behavioral Portfolio Theory* in 2 versions:

- a single mental account version which is the *BPT–SA*;
- multiple mental account version which is the *BPT–MA*.

The *BPT–SA* is constructed on the basis of *SFPT* and *SP/A*. The *BPT–MA* shows how investors segregate their portfolios into different mental accounts and ignore covariance among the mental accounts through giving different propose for each. The *BPT–MA* is actually a representation of *layer–investment* where every single layer has its own propose and associated with a particular aspiration level. The *BPT* is much complicated than the *MPT* and contrary to risk tolerance, the *BPT* has 5 influential variables:

q_s – fear degree;

q_p – hope degree;

δ – strength of fear relative to hope;

γ – strength of the desire to reach the aspiration level relative to fear and hope

A – aspiration level.

While Markowitz's mean–variance frontier is obtained by maximizing m for fixed σ, *BPT–SA* frontier is obtained by maximizing expected wealth $E_h(W)$ for fixed probability $Prob\{W < A\}$. When returns are normally distributed and no short sales are allowed, some optimal *BPT–SA* portfolios are on the mean–variance frontier.

The *BPT–SA* investors consider the covariance and integrate their investment into a single portfolio, but the *BPT–MA* investors are likely to split their investment because they overlook the covariances. Psychologically, they may see money in a different perception as it was predicted by Kahneman and Tversky (1979). This may explain why individuals with massive savings are likely to borrow for their other needs.

The generalization of all goal–based theories was made by **de Brouwer** in **2009**. He finds a similarity of the *BPT* with the postulates of Maslow (1943), the father of the **hierarchy of needs**. Maslow (1943) explains that the needs are built as a pyramid. After satisfying the basic needs, an individual demands satisfying the higher–order needs. De Brouwer (2009), showing the parallel with the such layers of the *BPT*, introduces the ***Maslowian Portfolio Theory* (*MaPT*)** where each layer of the *BPT* may be seen as a pyramid layer in the sense of Maslow (1943) with different investing goal. The central idea of the *MaPT* is that an average investor should keep a separate portfolio for each important life goal.

Additionally, the behavioral finance as a legitimate field started with **Shiller (1981)**. In 2003 Shiller issues a great overview of the behavioral finance's evolution through the decades. It was driven by 2 main aspects:

- the excessive volatility of early 80s that cannot be explained with the fundamentals of classical paradigms;
- the failure of the traditional financers to catch the stock market reality with their existing theories and models.

Those aspects allowed him to push the idea of market micro–efficiency against market macro–inefficiency forward. This means that one single stock movement has greater importance than a movement of the entire market.

In early 80s, the issue of excess volatility troubled the concept of efficient markets complete acceptance. The parameter of fundamental deviation diverges with the volatility observed in stock prices, meaning the finance gives wrong explanation of a stock value or an investor is not fully rational as described by the *Homo economicus* concept.

To explain the traditional finance mismatches Shiller (2003) lists several concepts:

- **The first concept** describes ***feedback models***. According to the concept, the stock prices are driven by attractiveness of success that was achieved by some investors rather than by a portion of new information. The success of some investors pulls public attention and subsequence increasing demand for specific stocks. Naturally, increased demand should push the prices up making them even more attractive. If the feedback is not interrupted, it may produce after many rounds a speculative ***bubble***. Shiller uses the term *bubble* to describe abnormal deviation of stock prices from its fundamentals and argues that the traditional paradigms cannot explain such phenomenon.
- **Another concept** turns to inability of ***smart money investors*** to eliminate any *noise* caused by sub–optimal decision making of ordinary investors. Theoretically, if irrational decision drives the ordinary investors to buy stocks, *smart money investors* have to act contrary to this decision and hence to sell them (and vice versa). Practically, this mechanism is possible only in a case when the *smart money investors* significantly

outweigh the irrational investors. Another practical behavior of the *smart money investors* is **"riding"** the price trend created by the ordinary investors. The *smart money investors* try to profit from a *bubble* contradicting the traditional theory.

In the following parts, next central behavioral theories and models will be discussed:

- Prospect Theory of Kahneman and Tversky (1979), which is a psychological and hence behavioral basis for decision making process under uncertainty.
- Typical behavioral models that concentrate on a specific psychological bias:
  1) Shefrin and Statman (1994) — Behavioral Asset Pricing Model (*BAPM*);
  2) Barberis et al (1998) — model of investors' sentiment;
  3) Daniel et al (1998) — model of investor's overconfidence and self–attribution;
  4) Hong and Stein (1997, 1999) — model of gradual information distribution.
- Szyszka (2009) — Generalized Behavioral Model (*GBM*).

1.3.1. Prospect theory of Kahneman and Tversky (1979)

Making a decision under uncertainty is not easy, but necessary. Our economic behavior is strictly depending on our vision of future. Individuals may expect for event outcomes and integrate it into their financial strategy, but many decisions are based on beliefs concerning the likelihood of uncertain events rather than on objective probabilities.

Samuelson (1963) criticizes the *Law of Large Numbers* introduced by Bernoulli in 1738. Bernoulli (1738) believed that enlarging a number of tosses may enclose a probability to mathematical expectation in the sense of money that is on a risk. Samuelson (1963) points out that enlarging of number of tosses obviously enlarges the risks so if one toss is unacceptable there for sure two tosses should be unacceptable and if every single toss is unacceptable so should be the sequence. This condition should be regulated by a win/gain ratio, but the classical *St. Petersburg paradox* does not offer it. In this sense, the Samuelson's colleague argues that he would suffer the 100$ loss more than joy of the 200$ gain, but betting 100 times may turn out the law of averages to the colleague's favor. Samuelson (1963) explains that thinking of enlarging of tosses does not lead to a better chance to win.

Tversky and Kahneman (1974) try to catch the way how individuals evaluate subjective probability for future uncertain events. They find that ordinary individuals solve complex tasks by cutting it to a number of simpler tasks. That is because of ordinary human is unable or even may not know how to use mathematics in order to complete a task. Instead, the judgment about uncertain event is made up by heuristics. The heuristics are useful, but in some cases, this may lead to severe and systematic errors. They distinguish 3 main most common heuristic groups: *representativeness*, *availability* and *anchoring* as can be seen in Figure 7:

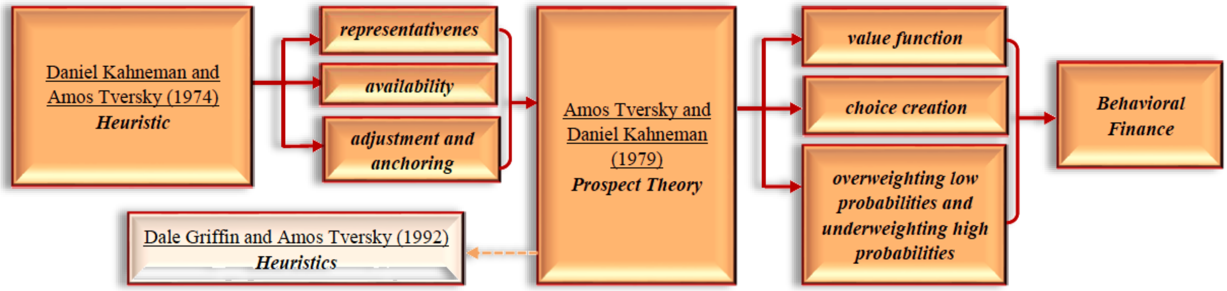


Figure 7. Prospect Theory as a platform for behavioral finance
Source: Own work

- ***Representativeness***: is linked to a similarity. Individuals evaluate their subjective probability and possible outcomes on a comparative base. Consequently, if *A* is similar to *B* than an individual evaluates a high subjective probability for event *A* originates or belongs from/to class *B* and vice versa. Here, individual's thinking process is based on matching stereotypes in his environment. Such heuristic completely ignores other influential factors for probability determination like sample size or prior probability that have greater influence on objective probability than an individual may think.
- ***Availability***: is associated with the ease of instances or occurrences which can be brought to mind. An individual may impact some phenomenon thinking about it as quite common assign to it high probability. That is because instances of large classes are usually recalled better and faster than instances of less frequent classes. The most explainable example of the heuristic is that if all of my friends have *A* for an exam, I may incorrectly think that there is a high probability to achieve *A*. Such heuristic harms the understanding of probability distribution.
- The heuristics of ***adjustment and anchoring*** are extremely sensitive to a starting point. The starting point may be given exogenously but could be a result or value of some partial estimation that is made by an individual himself. This initial value is adjusted to a final result, but the adjustment is close enough to the initial value. The phenomenon of relatively close adjustment to an initial position is called anchoring. The anchoring usually leads to miss–estimation because of unwilling of an individual to move from an initial value even if a true value is quite far from it.

Discovering and defining the heuristics let to Kahneman and Tversky (1979) investigate how individuals make their decisions when future outcomes and their probabilities are known to the individuals. They find that individual's observed preferences systematically violate all of the basic axioms of the subjective *EUT* in their actual decision–making behavior at least some of the time. They describe several psychological phenomena like *certainty effect* or *reflect effect* that contradicts the rationality of the *EUT*. In respond of their findings Kahneman and Tversky (1979) provide an alternative, empirically supported model for a theory of decision making under risk which is the *Prospect Theory*. This theory has 3 main characteristics that make it absolutely different from the classical approach. The first one is a way of choice creation. The *Prospect Theory* postulates 2 phases for a decision:

- the first phase is editing or *framing*;
- the second phase is actual evaluation of final result.

During the first phase, an individual prepares himself to the offered prospects dividing its complexity to simpler components and using the heuristics, described in Tversky and Kahneman (1974). In addition, during the same phase, individuals use:

- *coding* — labeling possible outcomes as gains and losses, but not as wealth or welfare;
- *combination* — reducing probabilities for identical outcomes;
- *cancelation* — similar to *independence axiom* of the *vNM* utility, i.e. identical parts of two prospects could be canceled in an individual's mind;
- *segregation* — riskless components are separated from risky components.

During the second phase, an individual evaluates the results of previous phase to a final actual value and the prospect of highest value is chosen.

The second characteristic is availability of a value function instead of a utility function and its general formulation is given by the next simplified formula:

$$V = \sum_{i=1}^{n} \pi\,(p_i) v(x_i), \tag{1.20}$$

where:

V – the overall or expected utility of the outcomes;

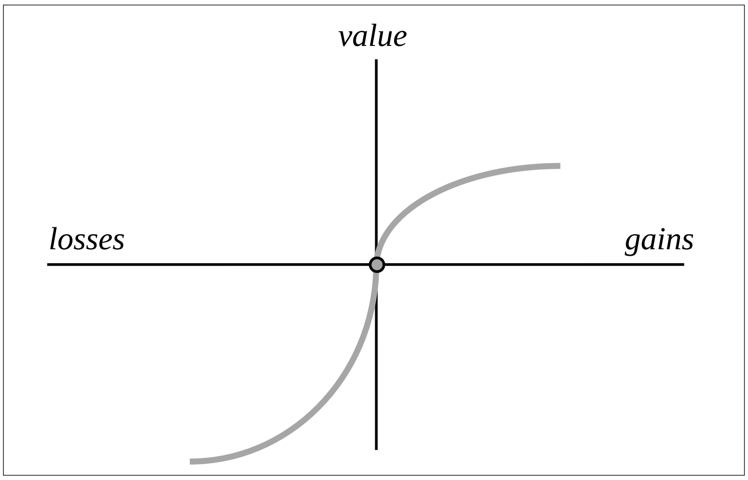
$(x_1, x_2 \dots x_n)$ – the potential outcomes;

$(p_1, p_2 \dots p_n)$ – the respective probabilities;

$\pi(p_i)$	–	the weighting probability–associated function;
$v(x_i)$	–	the value for a single outcome of the prospects.

After the second phase of a choice, an individual assigns a value V to the offered prospects which contains a weighting probability–associated function $\pi(p_i)$ and a value for a single outcome of the prospects $v(x_i)$. Kahneman and Tversky (1979) show that $\pi(p_i)$ is not a probability measure and the sum of weights for complementary probabilities typically falls below a unity, i.e. $\pi(p_i) + \pi(p_i) \leq 1$, while the sum of objective probabilities is always equals to 1. Meanwhile $v(x_i)$ reflects the subjective value of that outcome x_i. A value function V catches deviations from some zero point and shows how individual's judgment changes relatively to that point rather than in terms of final states or absolute value, like in a case of utility function. Hence a value function depends on two arguments: the asset position for a reference zero–point and the magnitude of positive/negative changes from that point.

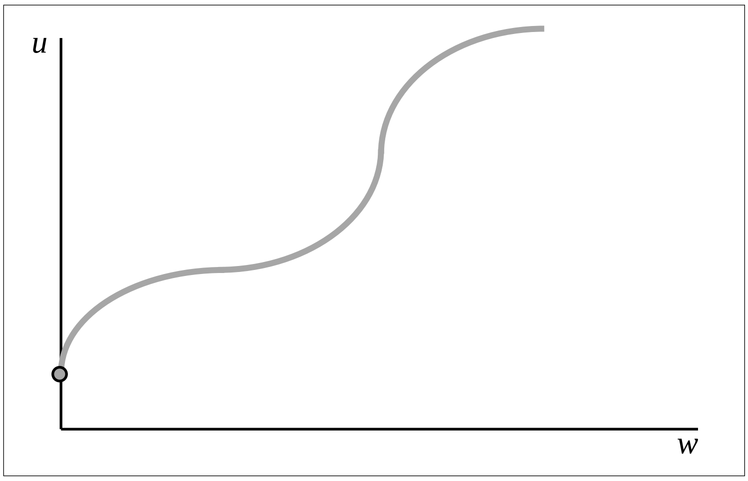
The third characteristic is associated with individual's reference about gains and losses and its subjective probability. A value function V has two domains and ***S*–shape form** as shown in the Graph 3.



Graph 3. A hypothetical value function
Source: Source: Kahneman, D., & Tversky, A.N. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47*(2), p. 279

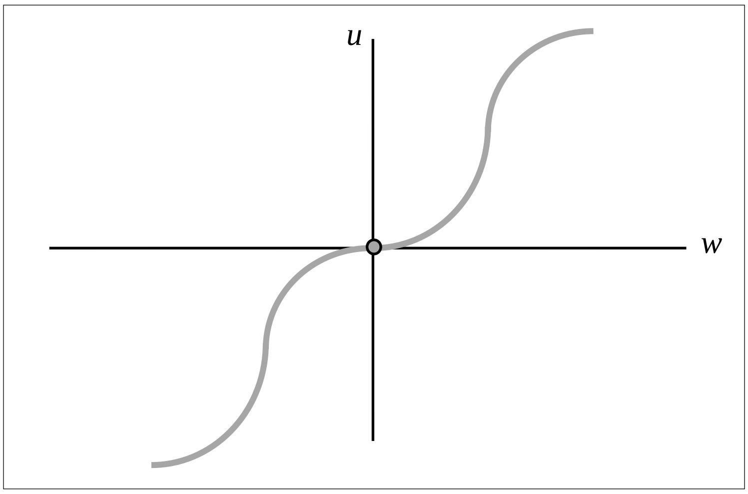
The first domain of the function describes gain references and it is completely positive. This part of a value function is generally concave and describes risk–averted behavior. The second domain is absolutely opposite to the first and describes loss references. This part of a value function is commonly convex and describes risk–taking behavior. Moreover, the slope of convex part of a function is steeper than the slope of the concave part meaning that a loss of 1$ harms more an individual than a gain of 1$ pleases him. For this reason, individuals take all possible opportunity to avoid a loss, even by taking extra risks. Note that a value function is steepest at the zero–point, but further flattens. Such form may explain the existence of *certainty effect*. From one side individuals prefer less, but certain amount of money instead of the offered prospect with potential higher payoff. From other side, the individual's willingness to take an extra risk increases when a loss is obviously close to be certain.

The idea of different references relatively to gains and losses was introduced by **Friedman** and **Savage (1948)** who propose an alternative for a utility function with concave and convex components similar to the value function of Kahneman and Tversky (1979), as shown in Graph 4. However, all of their function contains only positive values. This was the first attempt to describe the observed paradoxical individual's behavior, where the same individuals preferred to buy insurance policies and lottery tickets simultaneously. Also, they tried to explain within the function why an individual is risk–taker when he has more wealth buying a lottery ticket and risk–averted when he is poorer buying insurance policy.



Graph 4. Friedman–Savage utility function
Source: Friedman, M., & Savage, L.J. (1948). Utility analysis of choices involving risk. *Journal of Political Economy, 56*(4), p. 297

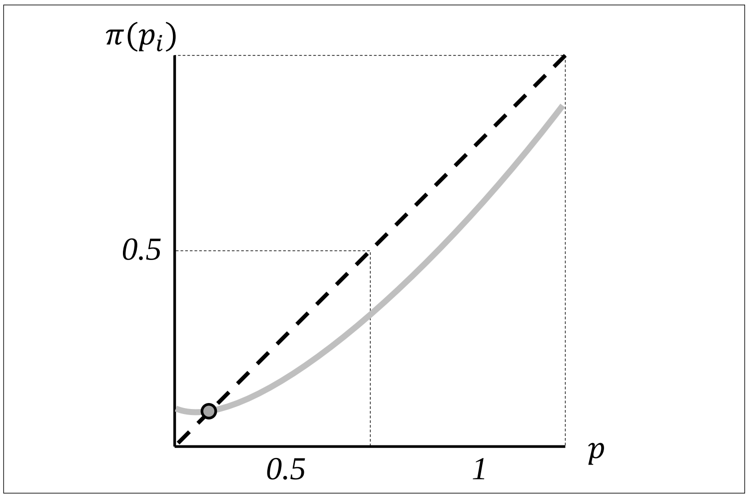
Markowitz (1952b) modifies the Friedman–Savage concept and suggests another alternative, which is *Customary Wealth Theory* (*CWT*) with three inflection points. He argues that problems of the Friedman–Savage utility are possible to eliminate if the first inflection point of the utility falls exactly at the individual's current wealth, as shown in Graph 5.



Graph 5. Markowitz's Customary Wealth utility
Source: Markowitz, H.M. (1952b). The utility of wealth. *Journal of Political Economy, 60*(2), p. 154

Markowitz (1952b) is the first to introduce the idea that individual's decisions are based on changes in wealth as well as on their total wealth and Kahneman and Tversky (1979) reach similar conclusions. However, both approaches have 2 principal differences:

- The first difference is a reference to a *total value* contrary to a *change in value*. Markowitz (1952b) argues that *total value* of wealth is important for an individual, but also a *change in value* makes sense. Kahneman and Tversky (1979) empirically proved that only *change in value* plays a role relatively only to a reference point.
- Another difference is connected to the subjective probability evaluation represented by the function $\pi(p_i)$ as shown in Graph 6. Kahneman and Tversky (1979) find that the individuals overweight low probabilities, but underweight high probabilities.



Graph 6. A hypothetical weighting function
Source: Kahneman, D., & Tversky, A.N. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47*(2), p. 283

1.3.2. Typical behavioral models

The traditional *CAPM* has a baggage of unrealistic assumptions that creates econometric anomalies and puzzles. Even normative economists realize that simplicity of the model obviously cannot suit the market reality. Hence, the extensions of *ICAPM* and *CCAPM* for the model were introduced, but all the extensions are exclusively on the normative basis. Since the *Prospect Theory* was published, alternative behavioral explanations for econometric anomalies rose up and **behavioral extensions for the *CAPM*** started to appear. Behavioral economists were especially motivated in the developing such models by observing the failure of the most normative models and by absolute failure of the traditional *CAPM* to fit the reality. However, first descriptive model came out from the psychological area (see Figure 8). Usually, a specific psychological bias lies in a basis of specific behavioral model, which was legitimized by the work of Kahneman and Tversky (1979).

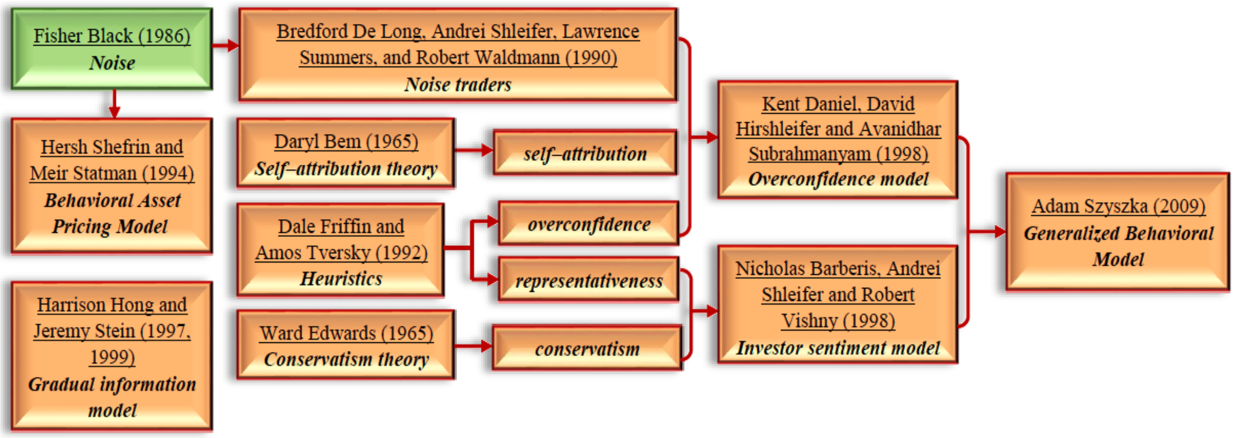


Figure 8. Typical behavioral models
Source: Own work

Black (1986) describes a situation where the information is imperfectly distributed in the markets. For some reason, the damaged information process is creating the ***noise***. The information arriving to individuals at the wrong time is also the *noise* and it is opposite to the information. Black (1986) theorizes that the *noise* is everywhere in the economy and often has no much deference with the information. For this reason, some traders may trade on the basis of *noise* if it was the true information. *Noise traders* obtain lots of information, which comes from technical analysts, economic consultants and stockbrokers and they falsely believe this information is useful to predict the future price of a risky asset. Since Black (1986) explores the problem from the normative point, the *noise traders* are normative with rational expectations and trading on a *noise* basis drives them out from the price efficiency. This may harm normative expected returns or give a push to speculative trend with abnormal returns for some particular asset. Black (1986) postulates that the *noise* creates an opportunity to get extra profit from the trading *information traders*. Therefore, it stimulates the financial markets and increases its liquidity. Otherwise, traders will hold individual assets with rare trading, because 2 sides with the same information are out from the willingness to trade. Therefore, the *noise* has 2–sided nature:

- the first side is its distortion nature that makes the markets inefficient and hence makes aberrant from the information–base (*EMH*) price;
- the second side is its motivating nature that creates profitability and liquidity for the *information traders*.

If Black (1986) refers *noise* trading to rational investors, the behavioral finance believes that origin of the *noise* is cognitive limitation of the investors, making them irrational in the sense of *Homo economicus*. Emotions–based decisions, limited access to the information, cognitive failure to evaluate a given situation, non–rational expectations — all these turns investors to the *noise traders* from the point of the *behavioral finance*. The traditional approach supposes that *noise traders* cannot survive in long–run terms because they lose their money and wealth to rational traders as buying high and selling low (Friedman, 1953). Contrary, behavioral approach supposes that the *noise traders* may obtain higher profit as they take higher risks relatively to the *information traders*.

The question of *noise traders*' survival was in the very center of **De Long, Shleifer, Summers,** and **Waldmann's (*DSSW* (1990))** research. They demonstrate that for plausible misperceptions, such *noise traders* as a group can not only earn higher returns than do rational investors, but also survive and dominate the market in terms of wealth in the long–run terms. The key point of the model is that the sentiment of the *noise traders* is unpredictable by the rational investors.

The *DSSW* (1990) model resolves the closed–end fund puzzle, discovered by Weiss (1989) where closed–end fund price is different from the net asset value that leads to irrational price, when a closed–end fund can trade at a premium at some times, and at a discount at other times. According to the model, *noise traders* influence the demand of closed–end fund shares and so the changes in discounts. If investors are optimistic, the funds are priced at premium and if *noise traders* are pessimistic, the funds are priced at large discount. In a case where *noise traders*' pessimism is constant, the risk of that *noise traders'* sentiment causing price changes to turn to systematic.

The first behavioral extension for the *CAPM* was made by **Shefrin and Statman (1994)** and essentially called the ***Behavioral Asset Pricing Model* (*BAPM*)**. According to the theory, all the investors are divided into 2 groups, where:

- one group contains only of the *information traders*, called single drive, who follows the traditional rules of the *CAPM*;
- other group contains the *noise traders*, called second drive, who are out of the *CAPM* rules and are easily to take a mistaken decision of subsequence cognitive failure.

The *BAPM* focuses on the market where *noise traders* and *information traders* affect each other and market efficiency question turns to a question of a group type dominance. Market is efficient only when the *information traders* are dominant over the *noise traders*, otherwise it is inefficient. The expected return of security is determined by the behavioral *beta*, which is the *beta* of tangent mean–variance efficient asset portfolio.

Shefrin and Statman (1994) assume that all the framework of the traditional approach of the *CAPM* holds, i.e. the risk premium is determined by *beta* and the distribution of returns on the *market portfolio* is normal. The only deference is the existence of the *noise traders* that drive prices away from the efficiency. From the other side, the *noise traders* create a positive conditional correlation between the abnormal returns and the *beta*. Also, they create excess volatility in the risk premium and in long–term interest rates. Shefrin and Statman (1994) found that the *noise traders* push down the market *beta* making the relationship between the security returns and new *beta*, distorted by the *noise traders*, weaker than those of the traditional *CAPM*. Therefore, the *BAPM* represents lower risk than the *CAPM*.

Through the model, Shefrin and Statman (1994) show that the *noise traders* may survive aside to the *information traders* even in an efficient market and may affect the market fundamental parameters. When the dominance of the *information traders* is obvious, the prices are efficient. In this case new information is no longer a sufficient statistic, but the old information continues to affect the market parameters, like volatility or risk premium, because it is still relevant for the *noise traders*. Shefrin and Statman (1994) show that efficient prices protect particular *noise traders*, since the *noise traders* involved in a trade only among themselves and their sole impact on the market is to generate excess volume. However, not every noise trading error is protected by market efficiency.

Another approach was introduced by **Barberis et al (*BSV* (1998))** in their **model of investor sentiment**. They developed the model in a respond to a huge amount of empirical evidence for the phenomena of underreaction and overreaction of stock prices. The evidences for **underreaction** are presented in Cutler, Poterba, and Summers (1991) that found a positive autocorrelation in excess index returns over horizons of between a month to a year or in Bernard (1992) that examines cross–section of stock section from the US and conjectures that market participants do not recognize the positive autocorrelations in earnings changes, and in fact believe that earnings follow a *random walk* that causes them to underreact to earnings announcements. The evidences for **overreaction** can be found in DeBondt and Thaler (1985), who discovered that portfolios of stocks with extremely poor returns over the previous 5 years dramatically outperform portfolios of stocks with extremely high returns, even after making the standard risk adjustments and the studies of DeBondt and Thaler (1987) with Fama and French (1992) who show the same findings. These phenomena are intuitively similar to the known *"good news and bad news"* phenomenon of Milgrom (1981).

The classical paradigm explains the phenomena in the terms of so–called *glamour* stocks with very high valuations relative to their assets or earnings that mostly are the stocks of companies with extremely high earnings growth over the previous several years which earn relatively low risk–adjusted returns in the future or in terms of *value* stocks with low valuation that earns relatively high returns. Fama and French (1992) argue that the *glamour* stocks are simply less risky while the *value* stocks are more risky. Though the *behavioral finance* tends to the psychological explanations using heuristics and argues that the information has much slower incorporation into the prices then it has been predicted by the *EMH*.

The investors are sensitive to the information and when an announcement is good, the investors are likely to predict higher average returns than the actual returns, creating underreaction of the stock prices. While an announcement is bad the investors are likely to demonstrate the opposite prediction, creating overreaction of the stock prices. *BSV* (1998) based their model on 2 works:

– Griffin and Tversky (1992), particularly on the ***representativeness*** heuristic, particular **the law of small numbers** when an individual pay too much attention to the strength of the evidence, but little attention to its statistical weight;
– Edwards (1968) who documented the phenomenon of the ***conservatism*** heuristic, defined as the slow updating of models with a new available information, i.e.

underweighting new information relative to priors.

The *BSV* (1998) model assumes that the *conservatism* should be associated with the underreaction which is defined as $E(r_{t+1}|z_t = G) > E(r_{t+1}|z_t = B)$ where: r_t is for return with $z_t = G, B$ is for *good* or *bad* news and shows that the average return on the company's stock in the period following an announcement of good news is *higher* than the average return in the period following bad news; while the *representativeness* should be associated with the overreaction which is defined as $E\left(r_{t+1}|z_t = G,\ z_{t-1} = G, \dots, z_{t-j} = G\right) < E\left(r_{t+1}|z_t = B, z_{t-1} = B, \dots, z_{t-j} = B\right)$ and occurring when the average return following not one, but a series of announcements of good news is *lower* than the average return following a series of bad news announcements.

The *BSV* (1998) build the model due to a representative risk–neutral individual where the true earnings process for all assets is assumed to follow a *random walk*. Despite this, the individuals have differed basis from a *random walk* to generate their forecasting of the future earnings. The individuals assume that at any time, the earnings can be generated only by 1 of 2 possible regimes:

- a **"mean–reverting" regime**, where the earnings are more mean–reverting than in reality;
- a **"trending" regime**, where the earnings trend more than in reality.

An individual believes that the generating earnings regime is given exogenously. For this reason, his goal is to identify which regime is valid and to react in accordance.

The mathematical model is built in the form of matrices which is the Markov' (1907) process in its structure. The model has 2 versions where the difference lies in the transition probabilities with $0 < \pi_L < 0.5$ and $0.5 < \pi_H < 1$. The first version (Model 1) describes the situation where a positive shock is likely to be reversed while the second version (Model 2), shows that a positive shock is more likely to be followed by another positive shock. The formal matrices are given in Table 1.1:

Table 1.1 Model 1 and Model 2 as two states of nature

Model 1	$y_{t+1} = y$	$y_{t+1} = -y$
$y_t = y$	π_L	$1 - \pi_L$
$y_t = -y$	$1 - \pi_L$	π_L

Model 2	$y_{t+1} = y$	$y_{t+1} = -y$
$y_t = y$	π_H	$1 - \pi_H$
$y_t = -y$	$1 - \pi_H$	π_H

Source: Barberis, N., Shleifer, A., & Vishny, R. (1998). A model of investor sentiment. *Journal of Financial Economics, 49*(3), p. 321

Model 1 was designed to respond the evidences of **short–term momentum** in stock return of Jagadeesh and Titman (1993) and delayed short–term responses of stock prices to earnings announcements of Ball and Brown (1968) and of Bernard and Thomas (1990). Model 2 was designed to respond the evidences of long–term reversal of DeBondt and Thaler (1985) and the returns to the contrarian investment strategies of Lakonishok, Shleifer, and Vishny (1994).

According to the *BSV* (1998), an investor tends to put more weights on Model 2 observing consecutive shocks on the same direction. Otherwise, he tends to Model 1 where the earnings are mean–reverting. If an investor assumes that Model 1 holds, he mistakenly thinks that the change is temporary. For this reason, a stock price should to underreact to the change in the earnings. When the investor's expectation is not confirmed by future earnings, stock prices show a delayed response to the earlier earnings. If an investor assumes (rarely) that Model 2 holds, he tends to believe that series of earnings may reflect a price trending and hence, mistakenly extrapolates such price trending, making the prices to overreact. Since the earnings are a *random walk*, the overreaction leads to reversal of long–term returns.

In the *BSV* (1998) model the price is given by expected present value of N_{t+j} future earnings, discounted with δ, which is possible to write as follows:

$$P_t = E_t\left\{\frac{N_{t+1}}{1+\delta} + \frac{N_{t+2}}{(1+\delta)^2} + \cdots + \frac{N_{t+j}}{(1+\delta)^j}\right\}.$$

In a case of a *random walk*, the price turns to be $P_t = N_t/\delta$, since $E_t(N_{t+j}) = N_t$ is valid. The model supposes that the investor does not use a *random walk*, hence the price should be deviated from its correct value. If so, the price is given as follows:

$$P_t = \frac{N_t}{\delta} + y_t(p_1 - p_2 q_t), \tag{1.21}$$

where p_1 and p_2 are constants that depend on the probabilities, while q_t is the specific probability that y_t is generated by the Model 1. The second part $y_t(p_1 - p_2 q_t)$ is the actual deviation from the fundamental value. Additionally, *BSV* (1998) provides sufficient condition for p_1 and p_2 to exhibit both underreaction and overreaction.

Additional reference to the under/overreaction is made by **Daniel et al (*DHS* (1998))**. Explaining the phenomena, *DHS* (1998) turn to 2 next biases:

- ***Overconfidence*** of Griffin and Tversky (1992);
- ***Self–attribution theory*** of Bem (1965).

The *DHS* (1998) model distinguishes 2 types of the investors:

- the first type is the *uninformed* investors that have no private information (signals) or late receiving of a public announcement that turns them to be *noise traders* and hence, risk averted;
- the second type is the *informed* investors that may have some information before other investors and they are assumed to be *risk–neutral*.

DHS (1998) assume that stock prices are determined by the *informed* investors that are driven by *overconfidence* and *self–attribution*. The *overconfidence* means that an *informed* investor is likely to overweight his private signals about a stock value which leads to overreaction, pushing a price to increase. From the other side, *biased self–attribution* causes an *informed* investor to underweight public signals, especially in a case when the public signals contradict his private signals, which leads to the underreaction. The overreaction to the private information and the underreaction to the public information create a short–term continuation of stock returns.

The *DHS* (1998) model is introduced for static confidence, as a single momentum, and for dynamic or time varied confidence. The prices are meant to set up in 4 stages, where all the random variables are meant to be independent and normally distributed:

(1) In the 1st stage — date 0, the investors have their initial wealth and the prices on their current present value.

(2) In the 2nd stage — date 1, the *informed* investors receive information which is private signal and start to trade with *uninformed* investors. The overreaction occurs due to overestimation of private signal by the *informed* investors;

(3) In the 3rd stage — date 2, the public announcement is available to all the investors and the *noisy* investors are involved into the trade. In the date 2 price correlation occurs reversing to the mean;

(4) During the 4th stage — date 3, the security pays dividends and consumption occurs.

During the static confidence, it is possible to set up equilibrium prices for every single date. The set of prices introduced in the model, by standard properties of normal variables of Anderson (1984), are as follows:

$$P_1 = E_C[\theta|\theta+\epsilon] = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_C^2}(\theta+\epsilon), \tag{1.22}$$

$$P_2 = E_C[\theta|\theta+\epsilon,\theta+\eta] = \frac{\sigma_\theta^2(\sigma_p^2+\sigma_C^2)\theta + \sigma_\theta^2\sigma_p^2\epsilon + \sigma_\theta^2\sigma_C^2\eta}{\sigma_\theta^2(\sigma_p^2+\sigma_C^2)+\sigma_p^2\sigma_C^2}, \tag{1.23}$$

$$P_3 = \theta, \tag{1.24}$$

where:

θ – the terminal value generated by risky asset;

ϵ – the error;

η – the noise;

σ_θ^2 – the variance the terminal value generated by risky asset;

σ_ϵ^2 – the variance of the error;

σ_p^2 – the standard deviation of the portfolio;

σ_c^2 – the expectation variance based on the *informed* investors' confidence beliefs.

$E(R_p)$ – the expected return of the portfolio.

During the dynamic confidence, the security price in each point of time is the expectation of its terminal value due to *risk–free* rate is zero and the *informed* investors are *risk–neutral*. Therefore the security price is given by:

$$\tilde{P}_t = E_C\left[\tilde{\theta}|s_1,\Phi_t\right] = \frac{(t-1)v_\eta\Phi_t + v_{C,t}s_1}{v_\theta+v_\eta+v_{C,t}}, \tag{1.25}$$

where:

Φ_t – the terminal value generated by risky asset;

s_1 – the error;

v_θ – $1/\sigma_\theta^2$;

v_η – $1/\sigma_\eta^2$;

$v_{C,t}$ – $1/\sigma_{C,t}^2$.

The *DHS* (1998) model shows that the overconfident investors lose their money on average. This is consistent with the intuition that the *informed* investors cannot trade with the *uninformed* investors. Also, according to the *DSSW* (1990) model, rational investors are not necessary to dominate the market. In a case, where risk averted investors are overconfident, they use the information effectively and hence may obtain higher expected profit than fully rational investors. The *DHS* (1998) model emphasizes that return predictability will be the highest relatively to firms with higher information asymmetry, making the inefficiency of the stock of small firms also higher. Though overconfident investors are not identified with a specific category of investors because even a small trader that presumably has less information, may be still overconfident. The *uninformed* investors of the model could be associated with contrarian–strategy investors.

While the *BSV* (1998) and the *DHS* (1998) models are based on a concept of representative individual with possible psychological biases, **Hong** and **Stein**[7] **(*HS* (1997, 1999))** assume that all the investors are **restricted rational** in the traditional sense, but with no psychological limitations and by this assumption they unify both the underreaction and the overreaction. In addition, positive feedback trading plays a central role in their model. They divide the investors between 2 groups:

- *newswatchers*, who make their forecasts based on signals that they privately observe about future fundamentals, hence do not condition on present or past prices;
- *momentum traders*, who condition on past prices changes, but their forecasts must be univariate functions of the history of the past prices.

The information is assumed to **gradually diffuse** across the population of investors and the model is given in 2 possible situations:

- the first situation is where the price formation occurs only with *newswatchers*;
- the second situation also includes *momentum traders*.

The following assumptions are used to define the model:

(A1) – *newswatchers* base their behavior on the ***Constant Absolute Risk Aversion*** **(*CARA*)** utility function;

(A2) – the *risk–free* rate equals to zero;

(A3) – the *newswatchers* to use present/past prices to forecast terminal dividend, $D_T = D_T + \sum_{j=0}^{T} \epsilon_j$;

(A4) – inability to forecast future price making the *newswatchers* unable to implement dynamic strategies.

With such assumptions, the price for any time period should be given by next equation:

[7] The HS model first time was issued in 1997 and the article version was issued in 1999, but already in 1998, Fama referred to it.

$$P_t = D_t + \left\{\frac{(z-1)\epsilon_{t+1} + (z-2)\epsilon_{t+2} + \cdots + \epsilon_{t+z-1}}{z}\right\} - \theta Q, \tag{1.26}$$

where:

z – the equal sized groups of the *newswatchers*;

ϵ_j – the dividend innovation;

Q – the asset fixed supply;

θ – the function of *newswatchers*' risk aversion.

The equation shows linear incorporation of new information into the price over z periods unconditionally to present and previous prices. If the condition is valid and the parameter θ is normalized to 1, then the equilibrium price, following a *random walk*, should be given by:

$$P_t^* = D_{t+z-1} - \theta Q. \tag{1.27}$$

Exactly as the *newswatchers* are assumed to base their behavior on the *CARA*, the same happens about the *momentum traders*. Contrary to the *newswatchers* the *momentum traders* in an attempt to forecast future prices use the information of present/past prices and their nature is closer to rational as in traditional sense. In this case, equation (1.25) could be rewritten as follows:

$$P_t = D_t + \left\{\frac{(z-1)\epsilon_{t+1} + (z-2)\epsilon_{t+2} + \cdots + \epsilon_{t+z-1}}{z}\right\} - Q + jA + \sum_{i=1}^{j} \phi \Delta P_{t-i}, \tag{1.28}$$

where:

j – the number of generations of the *momentum traders* at any time point;

ΔP_{t-i} – the time forecasting variable;

A – the constant;

ϕ – the elasticity parameter that has to be determined from optimization on the part of the *momentum traders* following by: $\phi = \frac{\gamma cov(P_{t+j} - P_t, \Delta P_{t-1})}{var(\Delta P) var_M(P_{t+j} - P_t)}$;

γ – the *momentum traders* aggregate risk tolerance.

In contrast to the equation (1.25), the part $S_t = Q + jA + \sum_{i=1}^{j} \phi \Delta P_{t-i}$ is so–called **aggregative supply**. Since the amount of traded asset is fixed and hold only by the *newswatchers*, they are the only source for the *momentum traders* to buy the asset that supply creates S_t.

In the model, the *newswatchers* produce underreaction, but never overreaction as the price adjusts slowly due to gradual diffusion of private information, which could be associated with the *conservatism*. As a result of aggressive response to good news through they know that the *momentum traders* are not involved at this stage, pushing the price to increase over next periods. This is so–called *front running effect*. When the *momentum traders* are *risk–neutral* and try to profit from such underreaction, it leads to a perverse outcome of acceleration of initial reaction of prices in the direction of fundamentals.

From the other side, the *momentum traders* produce overreaction. Through their inability to determine in what part of a momentum cycle they enter and inability to know if a price should increase responding to the news or as a result of past rounds of the momentum trade. When the *momentum traders* are active, they trade on the basis of past price changes, thereby generating momentum and causing prices to overshoot in the longer run, arbitraging away any underreaction left behind by the *newswatchers*. Also, they extrapolate the trend too far, reinforcing momentum and pushing price away from intrinsic value which leads to an eventual reversal in returns. According to the *HS* (1997, 1999) model, if underreaction exists in a short–run, then overreaction is necessary to occur in a long–run.

The *BSV* (1998), *DHS* (1998) and *HS* (1997, 1999) models are very central and typical in the behavioral finance. They have good successes in explaining some financial puzzles that traditional theories failed to resolve. The models share similar empirical successes, but also similar empirical failures (Fama, 1998). Fama (1998) criticizes the models arguing that they are good to explain the phenomena that they have been designed to explain. However, they miss the "big picture". The behavioral finance has no any fundamental approach contrary to the traditional finance.

An attempt to make a fundamental behavioral asset pricing model was made by **Szyszka (2009)** by generalization of all common psychological factors and biases into a single model, which is the ***Generalized Behavioral Model*** **(*GBM*)**.

Szyszka (2007) summarizes the studies on *behavioral finance* with the studies on cognitive psychology which allowed him to divide the behavioral factors into three crucial groups:

- The 1st group contains *errors in the processing information signals* (ε_1). The errors in the processing information signals lead to overreaction to bad news when ε_1 is negative and to underreaction while ε_1 is positive. This factor group is associated with the *anchoring* of Tversky and Kahneman (1974) or with the *conservatism* of Edwards (1965). Additionally, investors' unrealistic optimism may cause significant mispricing.

- The 2nd group contains *representativeness errors* (ε_2) with two most common phenomena: the *short series effect* and the *gambler's fallacy*. The *short series effect* takes place when an investor makes conclusions based on limited observations. The *gambler's fallacy* is a failure to determine a probability as a belief that possible outcomes are likely to be expected than other, i.e. the number of outcomes should be in a line with the probability distribution even in small samples. The positive ε_2 leads to the overpricing and the negative leads to the underpricing.
- The 3rd group contains *instable preferences* (ε_3). Here, based on the *Prospect Theory*, the investors make their decision due to changes in the value rather than due to the final value. If a possible value change subjectively better than the initial investor's wealth, an investor is likely to be risk averted and prefers to ensure his gain by selling the assets, creating the temporary underpricing ($\varepsilon_3 < 0$). Otherwise, an investor prefers to hold the assets that have worse position in a hope that with the time, the assets may have higher chance to back to the initial point. When an investor finds himself below the reference point, his risk–aversion degree declines what creates the temporary overpricing ($\varepsilon_3 > 0$).

The additional important factor is the market *ability to self–correct*, A. According to the *EMH*, even if any irrational activity presents at the market, the rational investors eliminate possible distortions using arbitrage and hence, reversing the prices to their fundamental value. It means that all possible behavioral mispricing meets resistance of rational investors that reduces mispricing effects. Though the *behavioral finance* assumes that the arbitrage is limited through existing of several risks and barriers, like noise trading risk, fundamental risk or implementation of costs and institutional or regulatory barriers. The variable intensity of activities of irrational traders who may temporarily cause even larger deviations of prices from the fundamental values, expose rational arbitrageurs to possible losses despite having established correct positions in accordance with arbitrage rules. Thus, the A value is a measure of the market's *ability to self–correct* and can be used as a measure of the market's informational efficiency. When $A = 1$, the market turns to fully efficient as predicted by the *EMH* while when $A = 0$, the behavioral distortion is maximal.

The aggregation of all 3 behavioral categories with market's *ability to self–correct* is given by the next formula:

$$\tilde{B}_t = (\tilde{\varepsilon}_1(x_t) + \tilde{\varepsilon}_2(x_t) + \tilde{\varepsilon}_3(x_t))(1 - A), \tag{1.29}$$

where:

x_t – the random event at moment *t*.

The main idea of the *GBM* is to take a fundamental price value and to measure an aberrant from observable market price, caused by behavioral factors. For this reason, Szyszka (2009) assumes a presence of 2 types of the investors, where one type is consistent with the *EMH* and the other type has psychologically driven heuristics and biases (Szyszka, 2009, p. 4). The point is that rational investors do not consider the activity of behavioral investors and account their influence as a random variable, associated with the random walk. The fundamental value is given by $\tilde{F}_t = \tilde{F}_{t-1} + \tilde{v}_t$, when $\tilde{v}_t$ is the independent random variable of new information flow. According to Fama (1965), the best approximation for the fundamental price is the observable market price, which should be given as follows: $\tilde{P}_t = \tilde{F}_t + \tilde{\xi}_t$, where $\tilde{\xi}_t$ is the independent zero–mean random variable, representing the same information flow as $\tilde{v}_t$ (Szyszka, 2009, p. 3). Taking the behavioral factors $\tilde{B}_t$ into consideration, leads the price to its behavioral deviate value as: $\tilde{P}_t = \tilde{F}_t + \tilde{B}_t + \tilde{\xi}_t$. From this, if $\tilde{B}_t > 0$, then the overpricing occurs. Otherwise, it is the underpricing or even its fundamental price. Implying the equation (1.29) into the $\tilde{P}_t$, leads the price to its final the *GBM* form, which is given as follows:

$$\tilde{P}_t = \tilde{F}_t + (\tilde{\varepsilon}_1(x_t) + \tilde{\varepsilon}_2(x_t) + \tilde{\varepsilon}_3(x_t))(1 - A) + \tilde{\xi}_t. \tag{1.30}$$

The *GBM* has an ability to explain several market anomalies like continuations and reversals of returns. In addition, it may explain the origin of excessive volatility by fluctuations in the intensity of the behavioral error $\tilde{B}_t$. Temporary intensification of behavioral factors can explain calendar anomalies and dispersion in the intensity of the errors among different markets or assets can be responsible for the manifestations of a violation of the law of one price. Finally, varied intensity of behavioral factors with respect to various asset classes may lead to *book–to–market value effect*.

1.4. Comparison of normative and behavioral approaches to capital asset pricing

Objectively, traditional way of thinking in finance has a long and solid history of over a hundred years, while the behavioral finance is young and still being under development. The traditional finance has been succeeding with developing fundamental models and testing them over decades, while the *behavioral finance* suggests some models that succeed to explain single market anomalies. The traditional fundamental models attempt to capture the theoretical optimal decision that follows maximum wealth for an individual, though the *behavioral finance* describes the decision processes and tries to model real–life choices. For the modeling, the traditional finance uses **wealth or utility function** that is necessary, but such function itself is strongly dependent on individual's preference system, which is one of the major disputable points between the normativists and the behaviorists.

The traditional concept of individual's preferences is based on **rationality**. It does not mean that an individual is deprived of the emotions, as some behavioral critics argue, but it means that during an economic decision–making process an individual should put his emotions aside; otherwise, his decision could be wrapped or distorted and hence, he will fail to obtain possible optimum. Rational individual is very smart. He is able to obtain all necessary information and able to use it the best way with maximal utility for him as suggested by the *EUT*, because he is self–motivated. Also, rationality assumes that an individual is interested to maximize his profits in a sense of financial market that is possible to count in the units of money or the units of utility. He is always consistent with his decisions and his preference system, i.e. a utility function. Moreover, rational individuals, using the *Bayesian updating*, are able to evaluate a probability and return distributions. Also, the traditional finance mostly sees individuals as risk averted. It assumes that individuals should berry less risk though some individuals could be riskier than others according to their personal utility function.

Contrary, the behavioral concept argues that individuals are limited. That is not due to the lack of education, but because of human nature. The natural limitations are a result of psychological biases and cognitive failures that prevent from an individual to be rational in the sense of the traditional concept. Another argument turns to self–motivated nature of individuals. The *behavioral finance* postulates that to an individual several goals in his life, therefore only profit maximization cannot be sufficient base for a decision making. As Kahneman and Tversky (1979) show, the individuals permanently violet the *EUT* axioms and they even unable to formulate their own utility functions. Instead, individuals use a **value function**, consisting of positive and negative domains, making individuals risk averted in its positive domain, but risk takers in the negative. Individuals demonstrate irrational behavior and hence, the traditional–sense optimum is not necessary to exist. Another highlighting made by Kahneman and Tversky (1979) is that individuals fail in evaluating the probabilities. Individuals tend to overestimate it on low levels and underestimate it on high levels.

Another disputable point is a concept of **risk**. The traditional finance is based on the *MPT* foundation that assumes association of a risk with price volatility. A rational individual should be interested in minimizing volatility magnitude by a diversification — buying different assets, and to maximize expected return. The compromise lies on an *efficient frontier* where a diversification is maximal. In order to obtain maximal diversification level, an individual should consider the correlation between the chosen assets. The *MPT* is a symmetric model, i.e. high positive volatility fluctuation is worse the same way as high negative volatility fluctuation. There are 2 global types of risks: systematic and unsystematic. The unsystematic could be eliminated by dividing the investment between *risk–free* asset and a *market portfolio*. The systematic risk never eliminates which is probably a correlation with a *market portfolio*. Therefore, the systematic risk is volatility of a market as a whole.

In contrast, the *Prospect Theory* suggests that risk is a feeling of suffer or enjoyment. Individuals have different attitude to gains and to losses: relatively to the same absolute value, a suffering of loss is higher than a joy of gain. For this reason, individuals prefer less, but sure gains, quitting fast out of their investment and pulling losses taking extra risks. Such behavior explains why portfolios of individuals contain a portion of poor assets that the *MPT* may not suggest to hold. Another point of the concept is that there is a different investment for a different life goal and hence, every single goal–investment has its own risks that may significantly vary from the risks of other goal–investments. The behavioral finance suggests that different individuals face toward different risks though the traditional finance assumes that all the individuals opposed the same risks.

Additional disputable point turns to a **market mechanism establishment**. The traditional finance believes that the markets are perfect. That is because of rapid incorporation of new information into the prices by rational individuals that are able to figure out of its meaningfulness and immediately to react, influencing the prices. Every individual has full easy access to all relevant information and information arrives to all the individuals at the same time. Since the individuals are rational they supposed to behave the same manner in different degree of reaction. This follows that price are not predictable and new information cannot be a basis to profit and no place to abnormal returns over the fundamental average.

Another market mechanism that the traditional finance believes is **arbitrage**, which is the principle of one price for one asset. In a case of deviation from the arbitrage, as a result of mispricing, an opportunity for extra profit is opened and then the rational investors react to close a price gap immediately. This follows that the prices are always on their **fair value** and do not allow to make to the investors any extra profits in long–run terms.

Oppositely, the *behavioral finance* refuses the idea of perfect markets. This means that abnormal returns and extra profits are possible and risk map changes over the time. There are several reasons to support such view:

- The first reason is that individuals are not identical. Therefore, their ability to understand informational signals is different due to psychological biases or due to other non–economic circumstances like a speed of intellectual data processing.
- Another reason is access to information. Akerlof (1970) shows, that information is asymmetric when insiders naturally have more information than outsiders. The information distribution is not uniform due to different timing or due to its gradual distribution.

The *behavioral finance* believes that prices are at least partly predictable. When irrational driven investors dominate the market, the prices are out of their **fundamental value**. Such phenomenon as *bubble* allows to predict the trend and to obtain extra profit as a result.

Also, the behaviorists assume that arbitrage does not hold in long–time period. The ability of arbitrage is limited due to negative shocks and lack of perfect substitution. The arbitrage is affected by *noise traders* and their continuously widespread irrationality. Additionally, it is affected by existence of transaction and implementation costs with taxation.

The traditional concept describes a perfect financial world while the behavioral concept attempts to incorporate the real life observations into economic models. The *behavioral finance* contradicts the traditional finance in every concept point. Therefore, it is reasonable to ask why both of the competitive theories exist side by side, if only one should describe the reality at least better than the other.

– The first reason is the lack of a solid evidence of one theory to refute the competitive theory, while both have enough evidences to support their own view. Economists are careful with theories replacement while they cannot be absolutely sure about the controversial theory.
– The second reason is that even if one theory will be completely approved, it does not mean that the competitive theory should be eliminated, because both work in

different dimensions. That means if, for example, the traditional theory has failed, its fundamental indexes will still be in use to indicate the direction of preferred market position. This may help to a policymaker to take a better decision about a market regulation. In a case, where the *behavioral finance* will fail, it is still being able to explain market anomalies that traditional finance cannot.

The theoretical background can be summarized with introducing a number of significant theoretical frameworks, touching different aspects of the capital asset pricing. However, only one issue is in a very heart of all the theories, which is searching for the answer only on one question: how an investor should react to a new flow of information with subsequent securities' price and return changes? Different approaches in attempt to answer the question distinguish between fundamental, behavioral and technical analysis schools. Figure 9 describes and summarizes possible scenarios of capital asset pricing theories. It demonstrates rational and behavioral explanation of the outcomes regarding the reaction of a representative investor. Further, in Chapter 2, tests for the theoretical models are presented. Results, conclusions and problems regarding to the methodologies are also introduced.

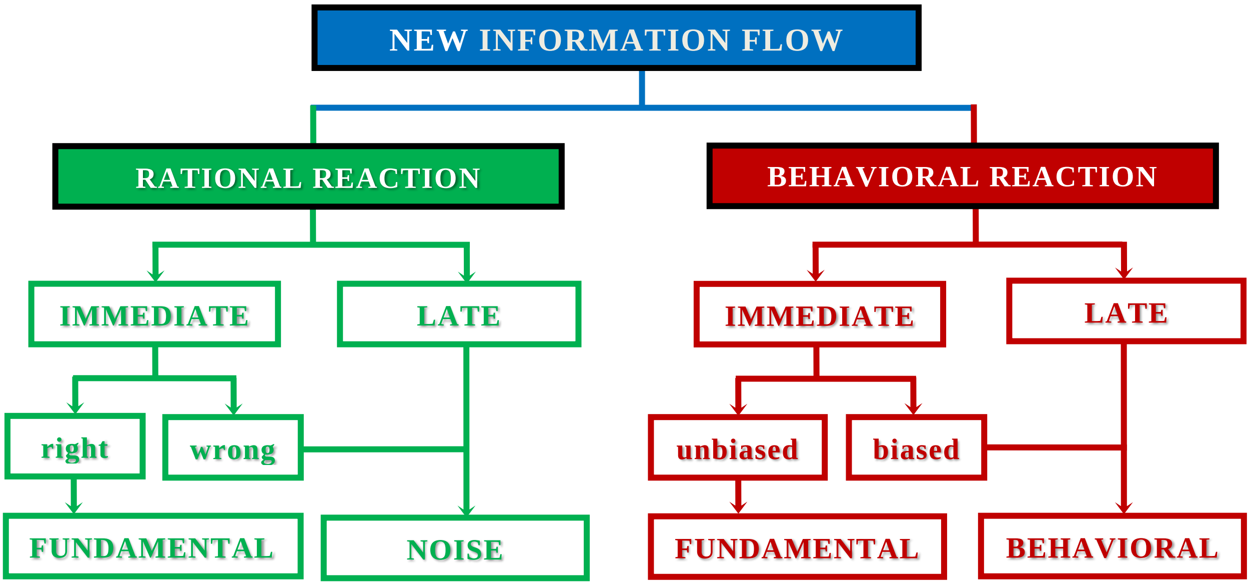


Figure 9. Reaction to a new information flow
Source: Own work

Chapter 2

Literature review on empirical tests of capital asset pricing

2.1. Tests of normative models

The normative or fundamental models are the basis for all financial models. Among them the most important are *CAPM* with its extensions, *APT* and multifactor models of Fama and French (1993, 1996, 2015). They are successful theoretically, but not necessary empirically. This chapter is a literature review of the empirical tests on capital asset pricing models. The special concentration is put on difficulties and improvements proposed in testing the models so they can be applied with a success to real information. The knowledge gained here is crucial for chapter three, where I present the universal capital asset pricing model and test it on real data.

2.1.1. Tests of CAPM

The *CAPM* is a very central, fundamental and most tested model in finance. It includes all of known normative ideas and findings until the model has been created. Cochrane (2005), summarizing a large body of empirical works, mentions 5 principles of the model that are the basis of the normative theory as a whole and turned to be a guiding line in testing the *CAPM*. Those principles are:

- market efficiency;
- arbitrage inability;
- inability to earn over average market returns;
- prices and returns are unpredictable (a *random walk*);
- the *Technical Analysis* is close to useless.

The empirical tests of the model are based on 3 implications of the relations between expected return and market *beta* suggested by the model:

- First is a linear relationship with expected returns and their *betas* and no other explanatory has influence on this relation.
- Second is expected return on a *market portfolio* is higher than those of the uncorrelated with the market assets.
- The last is expected return on the correlated assets with the market equals to the

excess return, i.e. return above the *risk–free* rate.

Most tests of these predictions use either cross–sectional or time–series regression or a combination of both. The cross–sectional approach is focused on predictions of an intercept and slope as a *beta*, regressing the average returns estimates of asset *betas*. The goal is to capture the intercept as the *risk–free* rate and the slope should express the excess return over the *risk–free* rate, $E(R_m) - R_f$. A popular **goodness–of–fit** measure used in many empirical studies is the cross–sectional R^2. As emphasized by Kan and Zhou (2004), R^2 is oriented toward expected returns.

Table 2.1 summarizes the aspects of the *CAPM* that are tested in the literature. More detailed description about them is presented later in the text.

Table 2.1 Tested aspects of the CAPM

ASPECT	RESEARCH	CONCLUSSIONS
Early tests	• Blume (1970); Friend and Blume (1970); Black, Jensen, and Scholes (1972); Fama and MacBeth (1973).	Testing and grouping portfolios is better than testing single assets.
	• Jensen (1968); Miller and Scholes (1972); Black, et al (1972).	Time–series regression findings violate traditional *CAPM*.
	• Stambaugh (1982); Gibbons (1982).	Black *CAPM* might be better than traditional *CAPM*.
	• Douglas (1968); Friend and Blume (1970); Black et al (1972); Miller and Scholes (1972); Blume and Friend (1973); Fama and MacBeth (1973); Stambaugh (1982); Fama and French (1992).	Positive flat relation between the *beta* and average returns. Its coefficients fall below the excess average market return, the intercept exceeds the average *risk–free* rate.
Market portfolio criticism	• Roll (1977); Stambaugh (1982); Kandel (1984); Lakonishok et al (1994); Fama and French (1996, 1998).	The *CAPM* is not testable since the *market portfolio*, does not exist. A market index proxy can be rejected.
Problems with riskless asset	• Roll (1970); Tobin (1958); Black et al (1972); Fama and MacBeth (1973, 1974).	90–day T–bill rates are serially correlated and therefore the returns and the prices do not follow a random walk. Intercept absolutely exceeds the T–bills proxy.
Anomalies and puzzles	• Basu (1977, 1983); Litzenberger and Ramaswamy (1979); French (1980); Banz (1981); Reinganum (1981); DeBondt and Thaler (1987); Bhandari (1988); Jegadeesh and Titman (1993); Amihud and Mendelson (1986); Shiller (1981); LeRoy and Poter (1981); Mehra and Prescott (1985).	Different anomalies violate the basis of the theory behind of the CAPM. However, it becomes weaker or disappear after discovering.

Source: Own work

Early tests

Already during the first tests it was revealed that the model works better with portfolios rather than with single assets to avoid measurement error problem and downward bias of a positive correlation in the regression residuals[8]. Diversified portfolios allow better estimation for *betas* since the *CAPM* explains asset and portfolio returns in the same manner. Hence, using portfolios of average returns on *betas* reduces the critical errors, but also reduces the statistical explanatory power. To avoid such problems, it is acceptable to group the assets on a basis of their *betas* from the portfolios that contain assets with lowest *betas* to those of the highest *betas*.

One of the first tests on the *CAPM* was produced by Lintner (1965) and later replicated by Douglas (1968). Lintner (1965) performs ordinary regression of $R_t = \alpha + \beta R_t + e_t$ and finds that the actual regressed values do not match the theoretical values. The intercept is too large relatively to the *risk–free* rate and the market excess return is statistically significant, but with lower value than expected. He concludes that the results seem to be in contradiction with the traditional *CAPM*.

Jensen (1968) is the first who used a time–series regression. In his work, he proposes some forecasting tools, including the *Jensen's alpha* (equation (1.9)). He rewrites the *CAPM* equation as follows and argues that such equation has a final form for the regression:

$$\tilde{R}_{jt} - R_{Ft} = \alpha_j + \beta_j[\tilde{R}_{Mt} - R_{Ft}] + \tilde{u}_{jt}. \tag{2.1}$$

Logically, if the excess return should be totally explained by the *CAPM*, then the intercept α_j for each asset should not be statistically significantly different from zero and so the error $\tilde{u}_{jt}$. Moreover, if a portfolio manager has an ability to forecast the prices then the intercept should be positive, since it represents the average incremental rate of return on the portfolio per unit time.

Another classical test implying a time–series regression is performed by Black et al (1972). Their method includes estimations on portfolios instead single assets. They demonstrate that

[8] Blume (1970) gives the original motivation for creating test portfolios of assets as a way to reduce the errors–in–variables problem of estimated *betas* as regressors. He argues that *betas* on the right–hand side of a regression give more precise estimates of factor loadings and for portfolios risk premiums. The same intuition for using portfolios as base assets in cross–sectional tests appears in Friend and Blume (1970), Black et al (1972) and Fama and MacBeth (1973). Their approach of grouping assets into portfolios has turned to be a standard empirical procedure today. The regression for cross–sectional estimation is based on the *SML* and given as:

$$R_i = R_f(1 - \beta_i) + \beta_i R_m + \zeta_{i,t}.$$

the empirical market line is linear with a positive trade–off between return and risk. With it the regression results show that the intercept is different from zero and in fact is time varying. They also find that the assets with low *betas* are likely to show positive intercepts and vice versa. Assets with the low *beta* may obtain higher return contrary to the theoretical *beta* of the same assets. Therefore, their findings violate the traditional *CAPM* as well as the conclusions of Lintner (1965).

Miller and Scholes (1972) discuss the statistical problem of all empirical studies of the *CAPM*. They propose another form for the time–series regression as follows:

$$R_t = R_{ft} + \beta_i(R_{mt} - R_{ft}). \tag{2.2}$$

Here, the intercept should be significantly different from $R_{ft}(1-\beta_i)$. The case when the R_{ft} is correlated with the R_{mt} and varies in time, automaticaly arises the problem of omitted variable bias that harms the *beta* estimation. Using historical data, they find negative correlation between R_{ft} and R_{mt} that intuitively is correct since rising of interest rates leads to a stock market declining. They also demonstrate that residual risk would act as a proxy for risk if *beta* had a large sampling error.

Another method to resolve the problem of correlation of the residuals in cross–section regressions is proposed by Fama and MacBeth (1973). According to their approach, it is better to produce the month–by–month estimation rather than to regress on monthly return averages and the regression should be done in 2 passes:

- The 1st pass is regular cross–sectional estimation;
- The 2nd pass is times–series measurement.

The time–series means of the monthly slopes and intercepts, along with the standard errors of the means, are then used to test whether the average premium for *beta* is positive and whether the average return on assets, which is uncorrelated with the market, is equal to the average *risk–free* interest rate. In this approach a variation through the months could be captured, but could be missed in the monthly averages approach. Their tests show that the intercept is much higher than the *risk–free* rate and hence, it may indicate that the classical *CAPM* does not hold. However today, **Fama–MacBeth method is the standard empirical procedure for testing the *CAPM*.**

Stambaugh (1982) employs a slightly different methodology. From the market model, he constrains separated equation for the intercept in a form $\alpha = k(1-\beta)$, where $k = R_{ft}$

under the standard *CAPM* or $k = R_{0m}$ under the *Black CAPM*. He performs a test, using Lagrange multiplier, and finds evidences to support only the *Black CAPM*. Gibbons (1982) adopts similar approach, but with maximum likelihood test instead. His test is based on an assumption that if the *CAPM* is true, then the constrained market model should have the same explanatory power as the unconstrained model; but if not, then the unconstrained model should have significantly more explanatory power than the constrained model. He performs the test that rejects both the traditional version of the *CAPM* and the version of Black.

Both the *CAPM* and the *Black CAPM* assume the *market portfolio* is a tangency point on the mean–variance *efficient frontier*. Therefore, all the differences in the expected returns should be explained exceptionally by the differences in the market *beta* and no other variable will have any explanatory power. In the Fama–MacBeth method, if all differences in expected return are explained by the *beta*, the average slopes on the additional variables should not be reliably different from zero. The hypothesis that market *betas* completely explain expected returns can also be tested using time–series regressions.

Gibbons, Ross, and Shanken (1989) and Kan, Robotti, and Shanken (2010) provide alternative tests of the validity of the beta–pricing relation. They propose to constrain a tangency portfolio by optimally combining the market proxy and the left–hand–side assets of the time–series regressions. Further, they compare the results with those of the construction by combining the market proxy and *risk–free* asset. This method helps to validate if the market proxy is really tangency portfolio and can be constructed by combining the *market portfolio* with the specific assets used as dependent variables in the time–series regressions. Thus, this is the test to validate whether the market proxy is on the frontier with minimal dispersion.

The early tests of Douglas (1968), Friend and Blume (1970), Black et al (1972), Miller and Scholes (1972), Blume and Friend (1973), Fama and MacBeth (1973), Stambaugh (1982) and even later works, for example Fama and French (1992) reveal positive, but flat relation between *beta* and average returns, however its coefficients fall below the excess average market return, which commonly proxies with US common index minus returns on 1–month T–bills and the intercept exceeds the average *risk–free* rate, which commonly proxies with US 1–month T–bills. The assets with the low *betas* are likely to demonstrate positive intercepts and the assets with high *betas* are likely to demonstrate the opposite trend. These tests and findings reject the classical version of the *CAPM*, but in general they are still consistent with the *Black CAPM*. The conclusions of the early tests are:

- the market proxies are on the *efficient frontier*;
- the market *betas* are able to explain expected returns;
- the risk premium for *beta* is positive;
- total rejection of the idea that the premium per unit of *beta* equals excess return over *risk–free* rate.

In addition, early tests reveal more success of the *Black CAPM* which came into consensus that this version of the *CAPM* model is a good description of expected returns.

Other popular extensions, like the *ICAPM* and *CCAPM*, are also tested. Merton (1980) shows that relation between the premium and the risk is indeed positive, though depending on estimation methods the results may vary and even lead to the opposite conclusions. Bollerslev et al (1988), Chou (1988), Kothari and Shanken (1995), Kothari et al (1995) and Scruggs (1998) provide early evidences supporting positive and significant risk–return relation of the *ICAPM*. Campbell (1993, 1996) introduces the most empirically successful theoretical frame to test the *ICAPM*, using the excess market return with discount rate news. In his tests a positive covariance of returns with volatility shocks predict a lower asset's return.

Mankiw and Shapiro (1986) compare the standard *CAPM* and the *CCAPM* specifications. They find that the standard *CCAPM* underperforms the *CAPM* with respect to individual stocks. Breeden et al (1989) comparing the empirical implications of consumption–based model with *market portfolio* based model, find that the standard *CCAPM* performs about as well as the traditional *CAPM*. These results are confirmed by Hansen and Jagannathan (1997) pointing out that the errors for both the *CCAPM* and the *CAPM* are rather large. Constantinides (1990), in his influential work, suggests that individual maximizes expected utility with an internal habit formation and then models it as exponentially decaying weighted average of past consumption rates. He demonstrates that habit persistence can generate the sample mean and variance of the historic consumption growth rate with a low exponent on the excess consumption term.

Market portfolio criticism

A very important part of criticism on the *CAPM* refers to the problems regarding to the subject of *market portfolio* or to its proxy. According to the famous Roll's (1977) critique, the *CAPM* cannot be tested since the *market portfolio*, the central component of the model, does not exist and there is no possibility to create it. In order to test the model, the *market portfolio* is forced to use as proxy, meaning the tests are probably reduced to figure out whether a proxy is efficient. If the investors are risk averse, then any mean–variance efficient portfolio would necessarily be linearly related to stock returns. While an investor's portfolio contains present value of labor, income and real estate, the stock market clearly not mean–variance efficient, making the *CAPM* untestable. Thus, no any test is acceptable except one, contains the real *market portfolio*. Therefore, in this sense the *CAPM* was never tested.

Stambaugh (1982) performs a number of tests, using different proxies such as common US stocks, real estate assets and government or corporate bonds. He finds that since volatility of stock returns dominate the volatility of expanded market returns, the expanding *market portfolio* proxies do not improve the results of the tests. He also finds that adding just few assets to the set of assets used in test, the linear relation can produce changes in inference since this is the nature of statistical inference. Even if the tested market index is inefficient with respect to the set of all the assets included in it, it might still be efficient with respect to some subsets of assets. Further, Fama and French (1998) perform the same tests including international assets in their proxies. They find that the *betas* are worldwide failed to explain the high average returns on the stocks with high *B/P* or *E/P* ratios. Kandel (1984) analyzes the testability of the mean–variance efficiency of a market index when the returns on some assets of the index are not limited to observe. The results demonstrate that bounding the market share of the missing assets and their expected returns are not sufficient for a valid test. A market index can be rejected if the variance of the missing assets is bounded and the missing wealth is small. As Lakonishok et al (1994) and Fama and French (1996, 1998) argue, the problem is that portfolios formed by sorting stocks on price ratios produce a wide range of average returns, but the average returns have no positive relation with the market *betas*.

Despite the criticism and in a response to Roll (1977), Fama and French (2004) argue that a proxy for the *market portfolio* indeed can be found because of requirement of mean–variance efficiency. Every portfolio on the *efficient frontier* potentially can be a *market portfolio* proxy, though if a proxy does not hold in the tests it won't hold in the application also.

Problems with riskless asset

Some critical arguments, emphasizing the problems of the *CAPM*, appeared already at the beginning of the 70s. Using the proxy of the 90–day T–bills, as acceptable in the academia, leads to variance and hence, to potential covariance with the market returns. Roll (1970) reports that T–bill rates are serially correlated and therefore the returns and the prices do not follow a *random walk*. He also finds that the serial correlation is not perfectly positive, which confirms the existence of some reinvestment risk. If the *risk–free* asset is correlated with *market portfolio* return, the *CML* would be convex. Tobin (1958) suggests that an asset's liquidity is crucial to the investors. For this reason, T–bills could be traded at a premium price leading to underestimation of the *CML* intercept but to overestimation of its slope. The short–term T–bills are significantly variable over time. Such variability could come from either the nominal rate of return or the return to compensate for expected changes in the level of prices. Hence, though the fixed coupon payment of the T–bills, their returns are not fixed. Another problem regarding the T–bills is that their rates are affected by interest rate control or by the money supply. Those rates are influenced by different macroeconomic factors, meaning that T–bills rates do not compensate for required level of the illiquidity and the expected inflation, but react to the economic stability.

Black et al (1972), based on the T–bills proxy, demonstrate that the estimated intercept of the model is different from the *risk–free* rate. This is because the intercept is depended on the *beta* of any asset where high–beta assets have different intercepts than those of low–beta assets. Fama and MacBeth (1973) calculate the actual risk premium and the predicted intercept over the period of 1935–1968 and also over different subperiods. Their results demonstrate that the intercept does not equal to the *risk–free* rate at any period. Later, Fama and MacBeth (1974) discover that the intercept absolutely exceeds the T–bills proxy.

Anomalies and puzzles

In the late 70s and in 80s of 20th century the explanatory power of *beta* was questioned since the researchers found more security characteristics with higher explanatory power than the *beta* as a result of well documented anomalies that were defined as empirical disparity between reality and theoretical model. The first anomaly is introduced by Basu (1977, 1983) and refers to the ***value effect***. He turns to different financial ratios and finds that some of them have higher explanatory power on the prices than *beta*. In particular, the firms with high earnings–to–price (*E/P*) ratios may earn positive abnormal returns. These findings were confirmed and extended by Reinganum (1981). Some subsequent works discover that positive abnormal returns may occur to the portfolios of stocks with high dividend yields (*D/P*) and with high book–to–market (*B/P*) values. The evidences for *D/P* effect first are provided by

Litzenberger and Ramaswamy (1979) and by Miller and Schools (1982). The evidences for *B/P* effect are documented by Statman (1980), Rosenberg et al (1985) or by DeBondt and Thaler (1987). Additionally, the researchers find a significant relation between the returns and value ratios that use cash flow containing accounting depreciation expense. Fama and French (1988) demonstrate that aggregate dividend yields predict subsequent stock returns. Bhandari (1988) finds that high debt–equity ratios demonstrate too high returns relatively to their market *betas*. Fama and French (1998) report that the *value effect* exists in a sample, covering 13 countries in the 20–years period. Chan and Lakonishok (2004) introduce the evidence that the value plays an important role in the security returns and Wu (2011) presents the importance of the *value effect* in China stock market.

Banz (1981) and Reinganum (1981) are the first to document anomaly of the ***size effect***. The *size effect* is the negative relation between returns and the market value of a common equity of a firm. Based on the *Jensen's alpha*, Banz (1981) and Reiganum (1981) show that small–capitalization firms earn higher average returns than they should earn, as the *CAPM* suggests, meaning that size coefficient has a higher explanatory power than the *beta* coefficient. Brown, Kleidon, and Marsh (1983) fully reexamine the study of Reiganum (1981) and confirm it with the same conclusions, but highlight that this effect is unstable over time. Keim (1983) reports a size premium of no less than 2.5% per a month in a broader sample when higher size *betas* stand for the small firms, but the difference cannot fully explain the return differential. Other evidence for the *size effect* for later sample period is presented by Lamoureux and Sanger (1989) though this time small firms had a lower *beta* than large firms. Fama and French (1992) conclude that *B/P* and size have the greatest explanatory power and argue that both are 2 variables that were omitted in the traditional *CAPM* thus it should be extended with those 2 additional risk factors (see Fama and French, 1993). Davis, Fama, and French (2000) find that the *size effect* is subsumed by the *value effect* in the earlier sample period. Kumar and Sehgal (2004) report strong influences of the *size effect* in Indian market.

During different periods, abnormal returns may occur comparatively to other periods, which is known as the ***calendar effects***. French (1980) observes the *weekend anomaly*. He notes that the average return on *S&P* composite portfolio is reliably negative on weekends. Smirlock and Starks (1986) find that the stock prices are likely to fall on Mondays where the closing price of Monday is less than the closing price of previous Friday. Keim (1983) and Reinganum (1983) demonstrate that abnormal return to small firms occur during the last week of December and first 2 weeks in January which is the *turn–of–the–year effect* phenomenon and may also refer to the *January effect*. The same works for the *turn–of–the–month effect*, documented by Agrawal and Tandon (1994). The prices are likely to increase in the last trading day of the following month, and the first three days of next month per every month in a year.

DeBondt and Thaler (1985) observe that stocks with low returns in the past 3–5 years have greater returns than stocks with high returns in the past 3–5 years. Stocks with prices on an upward (downward) trajectory over a prior period of 3–12 months have a higher return than expected probability of continuing on that upward (downward) trajectory over the subsequent 3–12 months, which is referred to the ***momentum effect***. Jegadeesh and Titman (1993) suggest a strategy of buying past winners and selling past losers simultaneously that allows generating some abnormal returns in a period of 3–12 months. Fama and French (1993), within their three–factor model, do not find evidences for overperformance in the long–term reversal strategy of DeBondt and Thaler (1985), but also cannot find explanation to the short–term momentum of Jegadeesh and Titman (1993) and Asness (1994). The *momentum effect* in long and short positions seems to be independent from market, size and value factors. Carhart (1997), using an extended version of three–factor model, demonstrates that the coefficient on the momentum factor is positive and statistically significant while it cannot be explained by the other three factors that has no rational risk–related explanation.

The liquidity anomaly or the ***liquidity effect*** is reported by Amihud and Mendelson (1986). They find a strong positive relationship between illiquidity and returns, but in Amihud and Mendelson (1989) they reach the opposite conclusion. This is because the investors require higher compensation for inability to trade. Since the illiquid stocks are out of daily trading, the bid–ask spread may rise, making the potential returns to excess the theoretical ones in both positive or negative directions. Eleswarapu and Reinganum (1993) duplicate the previous research with updated data and confirm the results, pointing it is mainly limited to January. Brennan and Subrahmanyam (1996), through the liquidity premium, find a positive return–illiquidity relation even after taking price, size and book–to–market factors into account. Their results generally support Amihud and Mendelson's (1986) findings, but do not support those of Elsewarapu and Reinganum (1993). Wang (1994) concludes that the volume is positively correlated with the absolute value of price changes and dividends. Datar, Naik, and Radcliffe (1998), Chordia, Subrahmanyam, and Anshuman (2001), Pastor and Stambaugh (2003), Liu (2006), Keene and Peterson (2007), Ho and Hung (2009), Lagoarde–Segot (2009) and

Jankowitsch, Nashikkar, and Subrahmanyam (2011) through different measurements provide empirical evidence to support the findings of Amihud and Mendelson (1986). Amihud (2002) confirms the study of Wang (1994) concluding that there is a significant relationship between liquidity and expected asset returns, proposing the illiquidity measurement of absolute return to dollar trading volume, *ILLIQ*. He finds a negative return–liquidity relation even in the presence of *size*, *beta*, and *momentum*, though Spiegel and Wang (2005) are failed to obtain any significant relationship between the expected return with both the bid–ask spread or the *ILLIQ* measurement after the controlling trading volume and the turnover.

The anomalies indicate that *beta* coefficient is not an appropriate risk measure. Though Schwert (2003) points out that most of the anomalies seem to disappear or become reverse or attenuate as they become known in the academic literature. In this sense, it is unclear whatever those anomalies are really profit opportunities existed in the past and arbitraged away by the investors or just a statistical aberrant that pulled attention of the researchers. The *size* and *value effects* have disappeared or dropped significantly since the initial publication of the papers that discovered it (Dimson & Marsh, 1999; Horowitz, Loughran, & Savin, 2000a, 2000b; Amihud, 2002). The same have happened to the *small–firm*, *turn–of–the–year*, *weekend* and the *dividend yield effects* that became weaker or lost its predictive power in the years after first documentation in the academic literature. However, the *momentum effect*, the *liquidity effect* and the influence of the *B/P* ratio are the most durable anomalies and they are still unresolved.

There is some evidence that despite the decrease in predictive power of discovered anomalies, some of them are not totally eliminated. For example, Malkiel and Jun (2009) report the existence of the *value effect* and *small–capitalization effect* observed on Chinese stock markets. The studies by Groot, Pang and Swinkels (2012), Fabozzi, Cakici, and Tan (2013) and Dimson, Marsh, and Staunton (2014) conclude that the *value effect* is statistically significant in emerging markets and the *momentum effect* is statistically significant for emerging markets and also for some developed countries. Raza, Shah, and Malik (2015) report *calendar effects* on the Pakistani stock market, where old calendar anomalies became much weaker or even disappeared after the documentation, but new *calendar effects* reappeared, replacing the old ones.

The standard *CAPM* has a massive baggage of unrealistic assumptions to simplify the model. These assumptions are necessary to highlight the only point of the capital market equilibrium through the connection between returns and volatility and prediction that an asset's risk premium that will be proportional to its *beta*. Naturally, it creates the anomalies. The anomalies can disappear or be present only in specific samples, missing in others. It is not a strong evidence that *beta* is unacceptable. Also the question about the *market portfolio* is still opened. However, the model and the theory are the most challenged by **puzzles** that describe a macroeconomic situation with much harder biases than unstable anomalies. Puzzles are questioning the fundamentals of the whole neoclassical theory.

The first significant puzzle, which is the ***volatility puzzle*** was discovered separately by Shiller (1981) with LeRoy and Poter (1981). Shiller (1981) compares the prices, as the traditional theory suggests an investor should expect relatively to the discounted dividend payoffs from the corporations, with the real stock prices observed on the market. LeRoy and Poter (1981) perform similar comparison, but they concentrate on bonds instead. If the theory is correct, then the prices should be smoother then in the reality. Shiller (1981) points out that if an investor is rational and the discount factor is constant, there is no explanation to the price hikes in the frame of standard theory and LeRoy and Poter (1981) reach the same conclusions relatively to the discounted bonds coupons.

The *volatility puzzle* is a significant challenge to the theory, but the most prominent is the ***equity premium puzzle*** discovered by Mehra and Prescott (1985). The equity premium is the excess return over the *risk–free* rate, which is essentially represented by the spread between a market index as a proxy for the market portfolio and T–bills as a proxy for the *risk–free* asset. Mehra and Prescott (1985) perform a number of empirical tests for the implications of Lucas (1978) seminal work, in which he argues that asset prices have a strong relation to the consumption in the context of a complete market equilibrium economy. In their tests, including the average level of the estimated real *risk–free* asset and the average level of the equity risk premium could be explained by the standard consumption–based model, Mehra and Prescott (1985) discover that the excess return is very variable and can exceed 19%, as in 1950s or can be around 0.3%, as in 1970s with the dissonance of 6.2% over the sample period. In the theory, it is reasonable to refer to the premium about 3%–7% per a year. In order to explain such significant measure of the risk premium, the coefficient of risk aversion that a representative investor has, should be extremely high, about 50, whereas in the reality the coefficient of risk aversion has been found experimentally by Kahneman and Tversky (1979), and it is about 2. From here, Mehra and Prescott (1985) argue that the real stock prices are excessively higher than it should be in a given risk level relatively to the *risk–free* assets (like bonds), regarding to the standard theory.

CAPM is the classical fundamental model that attracted financiers for decades. However, today after huge record of testing results, the model is recognized as inappropriate, yet gave a push to the researchers to look for better explanations of financial reality missed by the model. Even in present days the researchers still compare their results to those of the *CAPM*.

2.1.2. Tests of APT

The *APT* is a normative linear multifactor fundamental model generalizing both the *ICAPM* and the *CCAPM* extensions. Its main idea is that the returns correlate with macroeconomic variables through the impossibility of the arbitrage. In this sense, the investing opportunity set of the *ICAPM* and the consumption growth or *per capita* consumption of the *CCAPM* can be seen as **state variables** of the *APT*. Ross (1976) does not define the exact number of such variables though Roll and Ross (1995) emphasize 4 most important state variables that exclusively influence the return in a very long–run terms. However, every single scientist decides for himself which state variable should have a better explanation that led to controversial empirical results.

According to Shanken (1982) due to the lack of exact variable specification, leading to absence of potential estimated equation, the testing technique is complicated and difficult to perform. The empirical *APT* is absolutely different from the actual *APT* which harms testability. He emphasizes that the empirical tests are performed using actual assets, but not on arbitrary recombination and therefore, no the tests are related to the theoretical model were done. However, Dybvig and Ross (1985) in response to the Shanken's (1982) critique state that the *APT* is linked to a separation theory where asset returns are generated by a factor model. They also state that testing the *APT* on subset is typically valid. In the cases with biased testability the bias is towards rejection, so there is only little danger of spurious acceptance of the *APT*.

Following the idea that the pricing error represents diversifiable risk which is strongly bounded as the number of assets increases, a test of arbitrage pricing turns to a test of the behavior of the pricing errors as the number of assets increases without bound. Usually, the tests of the *APT* are based on assuming exact a *k*–factor model and testing it whether the intercept is insignificantly different from zero. Such test follows a 3–stage process, where: at the 1st stage, the returns equations are estimated to obtain factor loadings for the *betas*; at the 2nd stage, the conditional estimation for excess returns is needed and at the 3rd stage, the pricing condition is tested. The drawback of such approach is a difficulty of interpretation of the statistically derived factors in economic terms.

The model is tested with use of *principal components analysis* (*PCA*). Here, it is difficult to give an economic interpretation of the obtained results. Most of the studies are focused on the determining potential variables. Hence, it is possible to find 3 cluster thinking, where:

- the main one is the macro factor approach — macroeconomic arguments like *GDP*;
- the 2nd is the latent factor approach — statistical argumentation based on the factor analysis;
- the 3rd is the characteristic approach — financial arguments like *size* or *value*. Fama–French models deal with them.

Table 2.2 describes aspects regarding the *APT* tested in the literature. More details about them are presented in the following text.

Table 2.2 Tested aspects of the APT

ASPECT	RESEARCH	CONCLUSSIONS
Macroeconomic arguments	Gehr (1978); Roll and Ross (1980); Fogler, John, and Tipton (1981); Sharpe (1982); Chen (1983); Chen, Roll, and Ross (1986); Cheng (1995).	Define factor structure. There is a lot of disagreement about a number of factors included. However, it was found that the *APT* outperforms the *CAPM*.
Statistical arguments	Oldfield and Rogalski (1981); Chamberlain and Rothschild (1983); Ingersoll (1984); Connor and Korajczyk (1988); Garvett and Priestly (1997); Trzcinka (1986); Shukla and Trzcinka (1990, 1991).	Determine 5–7 factors for the *APT*. The first factor is the one most important and meaningful among other factors, suggesting that one–factor model can be sufficient.
Criticism of the model	Brown and Weinstein (1983); Dhrymes (1984); Dhrymes, Friend, and Gultekin (1984); Cho, Elton, and Gruber (1984); Gultekin and Gultekin (1987); Cho and Taylor (1987), Lehmann and Modest (1988).	Doubting if the classical five– or seven–factor structure is good. The number of factors may affect the results. The *APT* is rejected as a whole, but still able to explain effects which the *CAPM* remained uncovered.

Source: Own work

Macroeconomic arguments

The earliest empirical analysis of the *APT* is performed by Gehr (1978) through the variant of the cross–sectional approach where he applies a factor analysis. He estimates the vector of average risk premia within 3 common factors and finds that over the 30–year period only 1 has a significant premium. Over the three 10–year subintervals 1 factor is significant during the 1st subinterval, none of them during the 2nd and 2 factors are significant during the 3rd.

Such approach is adopted and further developed by Roll and Ross (1980) who extend the model up to 5–factor structure. In their classical study of the *APT*, they estimate factor risk premia and test the model restrictions through the 2–stage process and *Generalized Least Squares* (*GLS*) in the cross–sectional regressions. They discover that at least 3 but probably 4 of 5 common factors have significant explanatory power. They include the sample standard deviation of the asset as an instrument in cross–sectional regressions, where the estimate of the standard deviation is not predetermined, which may lead to the rejection of the *APT*. To keep the *APT* valid, other variables, like the total variance, should not have any influence on the expected returns. To resolve the problem, they use disjoint subsets to estimate the inputs that reduce the potential pseudo significance. Performing a regression for the expected returns derived using the 5 factors from their factor analysis, against the total variance of individual returns, they find that though variances and average returns are highly correlated, the variance is out of additional explanatory power for estimated expected returns. The same is discovered after correcting the problem of positive skewness in lognormal returns. On this basis, the authors conclude that the *APT* cannot be rejected.

Another testing method is introduced by Fogler et al (1981). They assign economic meaning to stock market factors and examine possible relation to the prices of capital in the bond market. Such relation was obtained. Sharpe (1982) finds as he calls 5 "common attributes[9]". In addition, he lists basic industries: capital goods, construction, consumer goods, energy, finance, transportation and utilities as "eight attributes representing sectors of the economy".

Chen (1983) compares the empirical characteristics of the *APT* and the *CAPM*. During the cross–sectional regressions of the average returns, he finds that correlation between the first factor, which stands for *beta* and the market index is highly significant and positive while the risk premia of the factors all together are not significantly different from zero. Thus, he finds that on average, the *APT* has higher predictive ability than the *CAPM* and it is able to explain some residual returns that the *CAPM* remained unexplained. He introduces 2 additional tests

[9] Dividend yield: "prior 12 months' dividends paid to common stockholders divided by the market value at the end of the prior month"; firm size: "the logarithm (to base 10) of the market value of the firm's equity at the end of the prior month"; stock beta: the slope coefficient from a regression of "the excess returns on a stock over the prior 60 months on the S&P stock index"; alpha: the intercept from the regression used to calculate the stock beta factor; bond's beta: the slope coefficient from a regression of "the excess returns on stock over the prior 60 months on the excess returns on long–term government bond returns. (As it appears in Cheng, A.C.S. (1993). *International arbitrage pricing theory: Empirical evidence from the United Kingdom and the United States* (Doctoral Dissertation, London School of Economics and Political Science (UK)).

based on empirical anomalies of total variance in assets returns and the *size effect* of the *CAPM* after removing the part of the return that was explained by the *APT* model. The results demonstrate that both, variance and size have significant explanatory power over the unexplained residual returns left by the *APT*. He proves that it is possible to add more factors to the 5 factors considered in the classical model, as long as it necessary and this is his biggest contribution in the *APT* study.

Cho (1984) performs a test to support the generating process behind the *APT*. Using the inter battery factor analysis of the US data to determine a number of factors, he divides the assets into 2 industry groups. He does not find significant variation among the industry groups with respect to the factors and concludes that a group size has no effect on the underlying factors of return generating process. Bower, Bower, and Logue (1984) perform a comparative analysis and find that the *CAPM* cannot explain the returns better than the *APT* since the *beta* is likely to change depending on market conditions and regulatory. Historic data cannot guarantee the accurate setting of the *beta* will hold in the equilibrium.

In their seminal work Chen et al (1986) propose to use 5 unknown, but principal obvious macroeconomic factors[10]. Through a version of the Fama–MacBeth (1973) technique, they use the factors which are correlated with the yield curve or things that investors may care about. These factors are chosen as common stock prices should represent the *PV* of discounted cash flows, while changes in industrial production level refer to a probability and other factors refer to the discount rate. However, they fail to obtain significant difference between future predicted and actual returns and could not explain the *size* and the *value anomalies*, though they conclude that the factors are good to explain the equity returns. They find that the industrial production, the changes in the risk premium and the twists in the yield curve are significant in explaining expected stock returns. More interesting result demonstrates that the market index has higher explanatory ability in the time–series variability of stock returns compared to the chosen economic state variables. Burmeister and Wall (1986) continuing the study of Chen et al (1986) conclude that variability of stock returns could be explained by unanticipated changes in certain macroeconomic variables. Hamao (1988) replicates the previous research in Japanese equity market and shows that changes in expected inflation,

[10] The factors are: the change in expected and unexpected inflation, the unexpected change in the term structure of interest rates, the growth rate of anticipated and unanticipated changes in industrial production, the unanticipated change in the risk premium and the changes in a stock market index.

unanticipated changes in the risk premium and unanticipated changes in the slope of the term structure appear to have a significant effect in the market. However, different countries have different financial and economic structures, which need to be estimated using different proxies and methodologies.

Comparative studies of Burmeister and McElroy (1988) conclude that the *APT* works better than the *CAPM*. They find that the premium on the inflation and on the slope of the term structure are negative and insignificant, contrary to the study of Chen et al (1986). Poon and Taylor (1991) applies the same technique for the UK stock market and show that variables have no influence on the stock prices the same manner as in Chen et al (1986). They conclude that the methodology of Chen et al (1986) inappropriately describes observed pricing relationship or other macroeconomic factors should be used.

The biggest contribution to the study of arbitrage theory comparing the UK with the US markets was made by Cheng (1995). In response to a lack of the economic interpretations of the factors within the original framework, he develops the canonical correlation analysis for security returns and macroeconomic variables. He reveals at least 2 prominent factors behind the return generating process. He also reveals that the returns are positively correlated to the longer leading indicators, money supply, government security price index and unemployment rate while lagging indicator and interest rate have a small negative correlation.

Comparative analysis within the data of the Australian market was performed by Faff (1992) using the **asymptotic principal components** technique. He concludes that the *APT* is better than the *CAPM* though both models are weak in explaining the monthly seasonal mispricing in Australian equities. Groenewold and Fraser (1997) within two–stage regression process obtain significance for short–term interest rate, the inflation rate and money growth rate in Australia. They also find the *APT* obviously has a better descriptive power than the *CAPM*, however both models perform inadequately in out–of–samples. Ataullah (2001) uses macroeconomic variables as common pervasive risk factors within the data of the Karachi Stock Exchange. Through the *Iterative Non Linear Seemingly Unrelated Regression* (*ITNLSUR*) method he specifies 9 macroeconomic variables as common pricing factors, where 4 of them have influence on the risk premium and in determining the market returns.

Dhankar and Singh (2005) using Indian stock data find that the *APT* has greater ability to describe the return generation process and to forecast returns, hence still be preferred over the *CAPM* in the Indian market. Febrian and Herwany (2010) examining the data from the

Indonesian market of the 3 separate periods support the *APT*. Their results show that excess return averages were consistently negative and the risk premiums varied over the observation periods. Cross–sectional results indicate that the regression coefficient of residual variance is significant meaning there must be more explanatory factors than a single *beta*.

Statistical arguments

Additional cluster thinking within the *APT* framework is the **latent factor approach** where the macroeconomic factors are replaced by statistical factors that should have influence on the returns. This approach was pioneered by Oldfield and Rogalski (1981) as a response to the criticism of infinitely large number of factors potentially involved in the *APT*. They assume a valid ex–post and ex–ante return model for both sets of securities. Using a 5–step procedure, they analyze the response of common returns to statistical factors. Their results confirm that the *APT* model is an appropriate specification of ex–post and ex–ante security returns. They discover that returns on bond portfolios are linearly related to at least 2 factor loadings and the multivariate tests are inconsistent with one–factor to seven–factor variants of the *APT* as descriptive models of the US T–bills market. Also, they successfully demonstrate that factor generating model has higher predictive power contrary to a market model.

Chamberlain and Rothschild (1983) and Ingersoll (1984) extend further the original *APT* and provide a unique approach, which is the *PCA*. They prove that the Ross' asset pricing theorem stills hold under the approximate factor model. Based on the approximate factor model, Connor and Korajczyk (1986) develop an alternative method called asymptotic principal components. Their results indicate that given a large cross–section, the first *k* eigenvectors of this cross–product matrix provide consistent estimates of the matrix of factor returns. Later, Connor and Korajczyk (1988) and Jones (2001) extend the procedure to deal with cross–sectional and time–series heteroskedasticity respectively. Garvett and Priestly (1997) based on the UK data analyze whether the returns have an approximate or an exact factor structure and which of them may have a better implication for the *APT*. They compare empirical effects of different assumptions about the factor structure that returns should follow and reveal 6 significant factors within the approximate factor structure. None of the factors was significant under the exact factor structure. Reisman (1988) demonstrates that if asset returns have an approximate factor structure, the *APT* is consequence of a mathematical theorem that is possible to derive by replacing the variables with different micro/macroeconomic factors.

Trzcinka (1986) focuses on the behavior of the eigenvalues of the sample covariance matrix as the number of securities increased and tests the ability of sample covariance matrices characterized as having *k* large eigenvalues. He finds that only the 1st and the one eigenvalue dominates the covariance matrix, meaning one–factor model might be enough to describe the process. He emphasizes that the number of factors should to increase when the number of stocks, included in the portfolio, also increases. Shukla and Trzcinka (1990, 1991) use the *PCA* and the maximum likelihood factor analysis to analyze the cross–sectional pricing equation of the *APT*. They find that the first eigenvector is much adequate measurement of risk in contrast to one–factor models or even to a five–eigenvector model. They describe that 1st eigenvector has a much higher correlation with the equal weighted *betas* than with the value weighted, confirming the Brown's (1989) theoretical argument that the 1st principal component is the equal weighted market index if the idiosyncratic risks are equal across the firms. Morelli (1999), focusing on the similar topic, compare principal component and maximum likelihood methods for extracting the factors on a basis of structural changes like a market crash in stock market returns. He concludes that the factors extracted from security returns within the *APT* framework do not suffer from a structural break or changes.

Criticism of the model

Early 80s of 20th century brought some criticism in address of the *APT*. Brown and Weinstein (1983) within a bilinear paradigm applies the special case of the *APT*, with pre–specified number of factors. Their results are in conflict with five– or seven–factor model of Roll and Ross (1980). They conclude only 3 factors represent the best the observed variation in the data. Dhrymes (1984) finds a significantly positive relationship between the number of factors and the number of assets in the groups which have a potential to explain the returns. His results indicate that the original methodology for testing the *APT* may not be the appropriate one. Next, Dhrymes et al (1984) emphasize that original method has serious flaws regarding to the ***rotation problem*** where 3–5 factors can be found by increasing the size of the groups analyzed. In the response, Roll and Ross (1984) argue that despite the *rotation problem*, tests of individual factor pricing are meaningful and there are many reasons for the number of non–priced factors to increase in parallel with increase of a group size. However, Cho et al (1984) demonstrate that the original procedure has a problem of factor comparability that overstates the number of factors, arguing that extra factors might be identified that would reflect *betas* influence.

Dhrymes, Friend, Gultekin, and Gultekin (1985b) apply a new testing method where unique variance measures play a role of explanatory variables. They use risk measures from the data of daily cross–section returns for the second half–period of 1962–1972 to explain the daily cross–section returns for the second half–period 1972–1981. Their results are extremely sensitive to the number of assets used in 2 stages of the *APT* tests. They demonstrate that unique risk is fully as important as common risk. The authors introduce a comprehensive set of tests of the implications of the *APT*. They discover that unique variance measures of risk mostly have a miserable part in explanation of asset returns that contradicts the theory. In addition, they identify the model relationship should be nonlinear between the expected rates of return and the measures of risk parameters. In parallel, they confirm that the intercept is the same across all groups which may indicate existence of *risk–free* or *zero–beta* rates of return, but significantly different from the T–bill proxy. Gultekin and Gultekin (1987) through maximum–likelihood factor analysis conclude that the *APT* is valid only in January. Cho and Taylor (1987) report the *January effect* and the *small–firm effect* on stock returns and confirming there is no significant statistics implying the *APT* across the groups.

Lehmann and Modest (1988) perform time–series tests for the *APT* with close–to–zero intercept restriction. They apply technique of *k*–factor mimicking portfolio analysis and zero–beta mimicking portfolio analysis to compare the performance with this of the one–factor model. They reject the *APT* as a whole, but they were not able to explain the dividend yield and variance effects which remained uncovered with the *CAPM*. Despite the rejection of the *APT*, they still argue that it is a better variant of the *CAPM*.

Middleton and Satchell (2001) analyze a possibility of using proxies instead the true factors and find that the *APT* is extremely sensitive to the number of the reference variables. In this case, the *APT* is out of testability. They argue that the number of the factors in the original model should be very large, otherwise it will suffer from inaccuracy. Reisman (2002) examines the model testability of *APT* in the light of approximate pricing under the assumption of finite number of assets and points out its violation of assumptions and impacts on testability.

After the exploratory factor analysis and prespecified macroeconomic factors approaches of 16 variables Iqbal and Haider (2005) obtain 9 factors that have the highest ability to explain the variances. They discover significance of only 2 factors in pricing the returns, which is too low to claim the stability of the explanatory power of the *APT*. Tursoy, Gunsel, and Rjoub

(2008) based on the monthly data of Istanbul, through the *OLS* method discover differences among the portfolios they constructed further concluding the variables have no significant influence on the stock returns.

Though the advanced financial thinking, the *APT* is difficult to be tested. The lack of exact variable definition makes the model disputable and out of uniformity leads to multiple theoretical and statistical models. The most famous ones used widely in capital asset pricing are the models of Fama and French.

2.1.3. Fama and French three–factor model (1993)

The Fama and French three–factor model has become the premier model within its class. It is seen as the extension of the *CAPM* and a private case of the *APT*. Fama and French (1992) were finally convinced that the traditional *CAPM*[11] *beta* has a weak explanatory power and payed their attention to discovered previously anomalies[12], which revealed that the *size effect* and the *B/P* ratio have the greatest explanatory power. Those types of risk were added to the basic *CAPM*, turning it to the famous three–factor model. With their model, Fama and French (1993, 1996) were successfully able to explain over 90% of the returns and argued that many of the *CAPM* average return anomalies can be captured with the model in addition. Their research results of the 25 portfolios, sorted on size and book–to–market equity, have become the benchmark in performance evaluation for the results of the upcoming new models.

First, Fama and French (1993) confirm that portfolios constructed to mimic risk factors related to market, *size* and *value*, all help to explain the returns for well–diversified stock portfolios. Further, Fama and French (1994) extend their conclusions to industries and Fama and French (1998) confirm it on international markets[13].

Lakonishok et al (1994) based on *BE/ME* ratio demonstrate that the abnormal returns are possible to obtain for common stocks supporting the three–factor model. As they demonstrate, the investors generate wrong expected returns and overpredict the returns of

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[11] In the original text of Fama and French (1992) they refer the *SLB* model (Sharp–Lintner–Black) to the traditional *CAPM*, though the model of Black has some differences. This is because they see the version of Black and the traditional *CAPM* both work at the same manner, though in Fama and French (2004) they fully distinguish the models and their empirical results.

[12] Stattman (1980), Banz (1981), Basu (1983), Rosenberg et al (1985), Bhandari (1988) and Chan, Hamao, and Lakonishok (1991) who discover a great role of book–to–market equity in explaining of cross–sectional average returns on Jappanese stocks.

[13] Europe, Australia and the Far East countries.

common assets with high *BE/ME* ratio while underpredict those of low *BE/ME* ratio. Lewellen (1999) also provides evidence that the Fama–French model is able to capture the variation in common stock returns.

Apart from the evidence from the US market there is a lot of research testing the model on data from other countries. Examples of such research are: Chui and Wei (1998) for Pacific–Basin emerging markets, Allen and Cleary (1998) for Bursa Malaysia Stock, Connor and Sehgal (2001) for Indian National Stock Exchange, Elfakhani, Lockwood, and Zaher (1998) for the Canadian markets, Chou, Li, and Zhou (2004) and Charitou and Constantinidis (2004) for Japan.

Within the Australian data Faff (2001) finds strong support for the Fama–French model, but with a significant negative, rather than the expected positive, premium to small size stocks. He hypothesizes that the results are consistent with evidence from other markets, on a reversal of the *size effect*. Gaunt (2004) finds that the *beta* is likely to be higher for smaller companies and those with lower *B/P* ratio. His study does not reveal a strong small firm effect, but the *BM/ME* effect increasing monotonically from the lowest to the highest book–to–market equity portfolios. O'Brien, Brailsford, and Gaunt (2012) based on *GRS* test conclude that the explanatory power of the Fama–French model is much better than that of the *CAPM*.

In contrast to the above studies, which are concentrated on Far East markets, Ajili (2002) compares the explanatory power of the three–factor model and the *CAPM* over the variation in common stocks from the French stock market. His results emphasize that the three–factor model has greater explanatory power than the *CAPM* and his findings support the risk–based view. Later within the same data, Ajili (2003) compares the risk–based view and the characteristics–based view. He finds that the market premium has very strong explanatory powers, while the *size effect* and value premium have a small impact on the excess returns. Bilinski and Lyssimachou (2004) perform similar study within Swedish data to support the three–factor model. Ferguson and Shockley (2003) using the equity only as a proxy for the *market portfolio* demonstrate that the characteristics correlated with a firm's relative leverage and relative distress have ability to explain the returns.

Based on the examples above, one may be convinced that the Fama–French model is a salvation of the problems associated with the capital asset pricing. However, despite the empirical evidence of the model performance, there is a lot of critique, arguing that the statistical methods applied in the tests are mostly wrong. Black (1993) made a concretely point that if researchers use any *market portfolio* proxy instead the true *market portfolio*, the *betas* are estimated with error when the stocks are seemed to have low *betas*, will on average have higher *betas* when the true *market portfolio* is used. He argues that the results presented by Fama and French (1993) may be based on a data snooping given the variable construction for the characteristics–based portfolios. Moreover, La Porta (1996) demonstrates that there is no evidence that the low expected growth stock is riskier than the expected high growth stock. Rouwenhorst (1999), examining emerging equity markets, failed to find that stocks with high *beta* outperform the stocks with low *beta*. Heston, Rouwenhorst, and Wessels (1999) examine the influence of *beta* and *size* on the market returns by replacing *BE/ME* ratio with a 1–period lagged market factor. Their results suggest that the model is failed to capture the variation in common stock returns. Frankel and Lee (1998), Dechow, Hutton, and Sloan (1999) and Piotroski (2000) argue that in portfolios sorted on the basis of price ratios, stocks with higher expected cash flows have higher average returns. Their findings could not be captured nor by the *CAPM* neither by the three–factor model. They refer the results to the irrationality of the prices in the sense of the *EMH*.

Daniel and Titman (1997) provide evidence, suggesting that the abnormal returns generated by the common stocks of firms with low *ME* and high *BE/ME* are not due to the common risk factors in returns, but through the characteristics of firms, rather than through the covariance structure of returns. While Fama and French (1993) consider *size* and *BE/ME* as risk factors, Daniel and Titman (1997) consider them as factors that reflect mispricing. In the response to the Daniel and Titman's (1997) results, Davis et al (2000) test the explanatory power of the model by employing portfolios constructed on past factor loadings in addition to *ME* and *BE/ME* ratios, when the characteristics–based view dominant over the risk–based view only in 1 sub–period. In their study Daniel, Titman, and Wei (2001) form zero–cost portfolios that were characteristic–balanced in parallel to the factor–balanced portfolios which could have a return of zero on average for common stocks where a firm's past *BE/ME* played a role of the factor loading. According to the findings, the risk–based view was rejected when characteristics–balanced portfolios were employed. Houge and Loughran (2006) based on the data regarding mutual funds with the highest loadings on the value factor are failed to obtain return premium.

Berk (2000) criticizes the method of sorting stocks. He shows that such method is biased toward rejecting whatever the pricing model is estimated in the 2nd sorting stage. Pettengill, Sundaram, and Mathur (2002), following the study of Berk (2000), find that problem of symmetric *beta* leads to an underestimation of the *size effect*. Bornholt (2007) emphasizes 2 main problems with the three–factor model:

- First, the method used by Fama and French (1993) for the construction of the factors that measure the *size effect* and the *book–to–market effect* is empirically defined and must be known ad–hoc.
- Second, there is a significant practical limitation when the implication should be done with a safe estimation of the sensibilities and the risk premiums for all 3 factors.

In addition, the factors of three–factor model are selected to explain the anomalies of past returns as historical accidents, but the explanatory power of the same factors for future expected returns is in doubt.

Qi (2004) based on the data from 12 US industry groups surprisingly conclude that the *CAPM* outperforms on an aggregate level, but with minor differences. Also, Bahl (2006) claims superiority of the *CAPM* in Indian market. Bartholdy and Peare (2005) emphasize that the Fama–French model is failed to capture the variation in common stock returns of firms quoted to NYSE over period of 1970–1996. Samer AM, Abdullah, and Izz (2010) examining emerging markets of Egypt, Jordan, Morocco and Saudi Arabia discover that only the *beta* has a significant power in the prediction of stocks returns while *size* and *value* have no.

Campbell, Hilscher, and Szilagyi (2008) find that the distress factor has no ability to explain the *size* and book or market factors, which leads to another anomaly, because the returns are found as significant in the wrong direction. Distressed firms have much higher volatility, market *betas* and loadings on value and small cap risk factors, so they have much worse performance in recessions. Similar patterns hold in all size quintiles, but are particularly strong in smaller stocks. They conclude that the distress did not generate a return premium, as suggested by the theory and hence cannot be a risk factor.

Cremers, Petajusto, and Zitzewitz (2013) argue that the value premium is overestimated in the Fama–French method because this methodology does not distinguish the differential impact of the value effects on small and larger sized portfolios since usually the *value effect* has a greater impact on smaller stock portfolios. Huij and Verbeek (2009) also report overestimation bias and in addition, underestimation of the momentum factors. Li, Brooks, and Miffre (2008) argue that the size premium could even be incorporated into the value premium and separation is not always possible to make.

Despite the solid argumentation supporting the three–factor model, there are enough studies to reject it. However, the model is the flagman model of the normative approach and the studies have never been finished.

2.1.4. Fama and French five–factor model (2015)

The five–factor model of Fama and French (2015) is a direct evolution of the three–factor model. The three–factor model is still being unable to explain several anomalies. Later, Fama and French (1996) admit the weakness of their model since it is designed to capture only a relation between *size* with *value effects* and average returns. Titman, Wei, and Xie (2004) demonstrate a negative relation between the increase in investments and average stock returns. A firm with higher level of capital investment exhibit subsequent lower returns. Novy–Marx (2013) demonstrates that gross–profits–to–assets ratio has similar explanatory ability as the *B/M* ratio regarding to average returns, when as a firm more profitable so higher the returns. Convinced by above arguments, Fama and French (2015)[14] introduce their five–factor model, updated with profitability and investment effects, which significantly increases the explanatory ability of the cross–section variation of returns.

Naturally, the earliest test for the five–factor model was made by Fama and French (2015), proving it has a better performance than the three–factor model on the US market. Further, Fama and French (2017) introduce an international evidence for a better performance of the five–factor model. Nichol and Dowling (2014), Abbas, Khan Aziz, and Sumrani (2015), Clarice and William (2015), Nguyen, Ulku, and Zhang (2015) and Chiah, Chai, and Zhong (2016) with Heaney, Koh, and Lan (2016) all confirm a better performance of the new model on the UK, Pakistani, Brazilian, Vietnamese and Australian markets respectively. Zhou and Zhang (2016) point that the five–factor model is able to explain the returns in China, but not so well as in the US market.

However, as the direct evolution, the five–factor model has inherited several problems from the previous version. The issue of low–beta is still being unresolved. Though Fama and French (2016a) claim that the low–beta anomaly is largely explained by their new model, Blitz and Vidojevic (2017) demonstrate that it is premature to make such conclusion, observing stronger mispricing for volatility than for *beta* meaning low–beta might be the dominant phenomenon. Additionally, exactly as the three–factor model, the five–factor model is still

[14] Firstly, the model was issued within a working paper in 2014, so several tests are done strating the same year.

ignoring the *liquidity* and *momentum effects*, which are much stable and important not less than *size* or *value* factors. As argued by Vidojevic, Hanauer, and Blitz (2017), the idiosyncratic momentum effect cannot be explained by the five–factor model, nor by the theoretical six–factor model where the five–factor is updated with *WML* factor of Carhart (1997), which is proposed by Jagadeesh and Titman (1993).

It is still unclear if adding two more factors really stays in a frame of economic rationale. Though the new model has a better explanatory ability, there is a doubt that adding two more factors is justified. Earlier, Fama and French (2008) conclude that investment and profitability are both not robust factors. It seems that there are factors which may be preferable over the ones chosen by Fama and French (2015). For example, Novy–Marx (2013) proposes an alternative (gross) profitability factor with better predictions of future firm profitability; Hou, Xue, and Zhang (2015) argue the five–factor model cannot to explain the net operating assets factor of Hirshleifer, Hou, Teoh, and Zhang (2004) and propose their own four–factor model instead. Further, Hou, Xue, and Zhang (2016) demonstrate that their model is able to explain every factor in the five–factor model. Other researchers like Pontiff and Woodgate (2008) or Linnainmaa and Roberts (2016) fail to observe profitability and investment premiums before 1970 or 1963. All this criticism demonstrates that adding factors to some basic model is not always efficient. However, in present days the five–factor model is considered as the most successful normative model.

2.2. Tests of behavioral models

The *behavioral finance* is much younger than the traditional finance, yet it may record several achievements. There are 4 typical behavioral models (*DSSW*, 1990; *BSV*, 1998; *DHS*, 1998; *HS*, 1997, 1999), that usually distinguish 2 groups of the investors. Every single group is biased by its own unique way and this is the theoretical basis for behavioral empirical tests structure. Those models are raised to deal with the *momentum* and *reversal effects*. Therefore, it is natural that support/rejection of those models are based on tests of the *momentum* itself. In general, the record of empirical evidence is controversial. There are 2 main reasons for such disagreement among the researchers:

(1) The normativists reject the idea of the *behavioral finance* and search for empirical invalidation of the behavioral theories as a whole, including typical behavioral models.
(2) Mostly, researchers perform tests for several typical behavioral models at once and

support some of them while reject the others, depending on the used methodology.

2.2.1. Testing typical behavioral models

Fama (1998) argues that typical behavioral models do well on the anomalies they are designed to explain though they misspoint a "big picture". With it he emphasizes that the models share the same success as well as the same empirical failure.

Table 2.3 summarizes the general conclusions for the tests of typical behavioral models, which are the basis for further researches based on exact sentiment proxies. The following text describes the results in more detailed manner.

Table 2.3 Tests for typical behavioral models

<table>
<tr><th>MODEL</th><th>RESEARCH</th><th>CONCLUSSIONS</th></tr>
<tr><td>DSSW (1990)</td><td>Sanders, Irwin, and Leuthold (1997).</td><td>There is no evidence that trader sentiment creates a systematic bias. Market returns using trader sentiment is not a characteristic of futures markets; Futures market returns at weekly intervals are characterized by low–order positive autocorrelation with relatively small autoregressive parameters.</td></tr>
<tr><td>BSV (1998)</td><td>Jagadeesh and Titman (2001); Bloomfield and Hales (2002); Chan, Frankel, and Kothari (2004); Doukas and McKnight (2005); Kausar and Taffler (2006); Alwathainani (2012).</td><td rowspan="2">The results are controversial. The BSV and DHS models are often seen as 2 sides of the same coin. Confirming one of them, automatically means rejection of the other.</td></tr>
<tr><td>DHS (1998)</td><td>Jagadeesh and Titman (2001); Chan, Frankel, and Kothari (CFK (2004)); Kausar and Taffler (2006); Fu (2016).</td></tr>
<tr><td>HS (1997, 1999)</td><td>Hong, Lim, and Stein (2000); Jagadeesh and Titman (2001); Chan, Frankel, and Kothari (CFK (2004)); Kausar and Taffler (2006); Doukas and McKnight (2005); Lin and Rassenti (2008); Bloomfield, Taylor, and Zhou (2009); Alwathainani (2012); Fu (2016).</td><td>Mostly, the information does not spread symmetrically. Bad news have slower diffusion since there is no real desire to announce them, hoping it can change during some period. By this, the HS model in general is confirmed.</td></tr>
</table>

Source: Own work

Sanders, Irwin, and Leuthold (1997), pioneers in testing trader sentiment within the *DSSW* (1990) model, apply it to futures market. Through the Fama–MacBeth cross–sectional regressions for examination of systematic bias existence in the futures prices with Cumby–Modest market timing framework for time–series predictability, they conclude:

(1) There is no evidence that trader sentiment creates a systematic bias;

(2) Market returns using trader sentiment is not a characteristic of futures markets;

(3) Futures market returns at weekly intervals are characterized by low–order positive

autocorrelation with relatively small autoregressive parameters.

Skinner and Sloan[15] (1998, 2002) focus on a question whether the differential returns between value and growth stocks are driven by a large asymmetric response to adverse earnings news in growth stocks. They use a sample of quarterly earnings forecasts from the *Institutional Brokerage Estimate System* (*I/B/E/S*) historical database. They report that an asymmetrically large negative price response to negative earnings surprise associated with value and glamour stocks, which is caused by expectational errors relatively to the future earnings performance. The market reacts more strongly to bad news for both types of firms. They argue that the phenomenon can be explained with psychological biases, appeared in *DHS* (1998) or *BSV* (1998) where the growth stocks gradually become overpriced after a series of consistently good earnings reports, but reversed after earnings disappointments since investors' expectations are highly optimistic.

Bloomfield and Hales (2002) perform 2 experiments to validate the regime–shifting beliefs introduced by the *BSV* (1998) model:

(1) For their first experiment, 38 MBA students were invited and asked to set a price according to 16 constructed graphs.

(2) During the second experiment, the same students were asked to set a price for constructed a single 80–period sequence, when a half of the participants saw a regular sequence and the other half saw the mirrored image.

They find that the 1st experiment captures investors' underreaction and overreaction, as predicted by the *BSV* (1998) model. The 2nd experiment reflects direct *BSV* (1998) prediction about investors' expectation relatively a single sequence to switch between trending and mean–reverting regimes. However, Alwathainani (2012) performs a test for the conservatism effect of the *BSV* (1998) model. His results are inconsistent with the *BSV* (1998) predictions while partly support private information models of the *HS* (1997, 1999) and the *DHS* (1998). His findings indicate overreaction of market securities to extreme information signals rather than cautiously responding to the new information.

Hong, Lim, and **Stein (2000)** perform a test to validate *HS* (1997, 1999) model. Following Jagadish and Titman (1993) methodology, they build size fixed momentum strategies based on the data from 3 sources[16], choosing stocks with low analyst coverage as a proxy for the

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[15] Firstly, their study appeared in 1998 as unpublished working paper, further, it was converted into the article.
[16] The data on stock returns and turnovers, analyst coverage and option–listing data is tooken from CRSP monthly

slow diffusion of news. They report that the *momentum effect* is particularly strong in stocks with low analyst coverage and among small firms and that information distribution not only gradual, but also disproportional where the loser stocks take longer to be fully reflected in prices as "bad news travels slowly". It was found that when the news are good, the analysts make effort to inform the investors as quickly as possible, while the news are bad, the analysts prefer not to hurry with the announcement, making the information diffusion much slower. Therefore, the losers have higher sensitivity to some particular news that leads to higher volatility persistence. The authors conclude that such phenomenon is the confirmation for *HS* (1997, 1999) model. Xiuqing (2008) replicates the previous study and claims that his findings strongly contradict the theory of *HS* (1997, 1999), but support the *DHS* (1998) model. Doukas and McKnight (2005) focus on validation of the *HS* (1997, 1999) and *BSV* (1998) models using previous methodology and dispersion in analyst forecasts as a proxy for the weight of information respectively. They confirm the findings of Rowenhorst (1998), as average stock returns are related to past performance, support the *HS* (1997, 1999) model, as momentum strategies work better in stocks with low analyst coverage that indicates the findings of Hong et al (2000) are not merely due to chance and support the *BSV* (1998) model, as analysts' forecast dispersion is inversely related with the profitability of momentum strategies. Bloomfield, Taylor and Zhou (2009) provide 3 laboratory experiments, based on the *HS* (1997, 1999) methodology. Their results are completely consistent with the *HS* (1997, 1999) predictions. However, Israel and Moskowitz (2013) fail to obtain evidence for decreasing in momentum returns with firm size, concluding that results of Hong et al (2000) are obtained only due to the sample specification.

Chui, Titman, and Wei (2003) report a significant relation between overconfidence and country momentum. In addition, they found that REITs in the US demonstrate weaker momentum in the period of slower information diffusion and stronger momentum in the period of more valuation uncertainty. Their findings contradict the prediction of the *HS* (1997, 1999) model while support the *DHS* (1998) model. Fu (2016), as it implies directly from the title of his article, performs a test for explanatory power of 2 behavioral models (*HS*, 1997, 1999; *DHS*, 1998) for price continuation in the Taiwanese market. Following the previous methodology, he suggests no support for *HS* (1997, 1999) and *DHS* (1998) theories, arguing

stock combined file, I/B/E/S historical database and Options Clearing Corporation respectively.

although the price continuation is a global phenomenon, the sources may be a market specific.

Jagadeesh and **Titman (2001)** continue to examine the *momentum effect*, they discovered in 1993. The authors see 2 controversial explanations of behavioral models (*BSV* (1998), *DHS* (1998) and *HS* (1997, 1999)) against rational argumentation of Conrad and Kaul (1998), hypothesizing the profitability of momentum strategies comes from cross–sectional variation in expected returns, but not from predictable time–series variations in stock returns. They perform a test through the returns of the winner and loser stocks in the 60 months following the formation date, comparing it with Fama–French (1993) portfolio performance as a benchmark. According to the results they support the behavioral explanation for the *momentum effect*, completely rejecting the rational argumentation. Though the authors emphasize that their results should be tempered with caution since they capture a strong evidence of return reversals for small firms and much weaker for large firms. Hong, Lee, and Swaminathan (2003) examine *momentum effect* in several European as well as far eastern markets. They argue that their results are most consistent with *BSV* (1998) and *DHS* (1998) models, finding that analysts underreact to past information in all countries.

Statman, Thorley, and Vorkink[17] (2004, 2006) fill the gap in research on investor overconfidence, applying *vector autoregressions* and associated *impulse response functions*. They report significant relation between trading activity and past returns, which is consistent with *DHS* (1998) model hypothesis. For their study 3 main findings:

(1) First, they find a statistically and economically significant positive relationship between market turnover and lagged market returns, which is consistent with the overconfidence.

(2) Second, they report that positive individual security turnover responses to both lagged own security returns and lagged market returns, which is consistent with the disposition effect, confirming investor overconfidence.

(3) Third, the relationship between returns and turnover is stronger in small cap stocks.

Jiang, Lee, and Zhang (2005) focus on *information uncertainty* (*IU*) as a sentiment drift and its involvement in predicting cross–sectional stock returns. They examine the potential relation between the information uncertainty and *Post–Earnings–Announcement–Drift* (*PEAD*). In their study, they obtain that stocks with higher *IU* level earn lower returns over the

[17] First working paper was issued in 2004, but further was converted into article.

succeeding 6–month period. They capture a positive correlation between overconfidence and arbitrage costs, which produce lower mean returns and greater momentum returns. High momentum returns are also present in small and young firms, firms with high trading volume turnover, high return volatility and low duration. They conclude that the results are consistent with the *DHS* (1998) theory. Zhang (2006) continues the previous study and completely confirms its findings, which are consistent with Chan, Jagadeesh, and Lakonishok (1996) and the *DHS* (1998) theory of increasing psychological biases with higher uncertainty.

Chan, Frankel, and **Kothari (*CFK* (2004))** examine the predictions of 4 typical behavioral models (*BSV*, 1998; *DHS*, 1998; *HS*, 1997, 1999; Mullainathan, 2001). They find no evidence to support any behavioral model, but structural uncertainty of rational models. There is no any relationship between the sequence of past accounting and future returns and hence is unlikely to bias investors' consensus expectations. Daniel (2004) responds to *CFK* (2004), introducing alternative interpretation of their findings. He points that though in general behavioral models are designed to answer the same propose, in particular each of them has different explanation approach and hence empirical implications. The most prominent is the opposite implications of the *BSV* (1998) and the *DHS* (1998) models. This automatically means that if one of the models has potentially wrong implication, then the opposite should has true implication.

Kausar and Taffler (2006) perform a test for theoretical predictions of under/overreaction of *BSV* (1998), *DHS* (1998) and *HS* (1997, 1999) models. They demonstrate evidence in support only of *DHS* (1998) theory. Kothari, Lewellen, and Warner (2006) examine the *PEAD* to obtain a connection between market returns and aggregate earnings surprises. They perform a test for Bernard and Thomas (1990), *BSV* (1998) and *DHS* (1998) (indirectly) models and for the correlation between earnings growth and movements in discount rates. Their results suggest inconsistency with all examined behavioral theories, but with the rational theories. Lin and Rassenti (2008) concentrate on potential source of under/overreaction and examine 4 behavioral models[18], adopting the *HS* (1997, 1999) framework. They find that the *DHS* (1998) model predictions are not confirmed, the *HS* (1997, 1999) mechanism for under/overreaction and overreaction cannot explain the drift pattern of prices and the *BSV* (1998) is the only suitable model with the inertia price patterns.

[18] BSV (1998), DHS (1998), HS (1997, 1999) and Frazzini (2006).

2.2.2. Tests of sentiment proxies

The main argument of the behaviorists is that the sentiment may permanently affect the fundamental prices, creating limited arbitrage. Since sometimes the arbitrageurs or "smart money" fail to resist the phenomenon, contradicting the suggestion of the normative theory, the sentiment turns to a *systematic risk*, expressed in additional volatility (Brown, 1999). For this reason, the question of existence of a relationship between investor sentiment and stock returns is a central empirical issue in the *behavioral finance* literature.

In the Table 2.4 are summarized acceptable proxies that further will be explained more detailed.

Table 2.4 Sentiment proxies

TESTED PROXY	RESEARCH	CONCLUSSIONS
Closed–End Funds Discount (CEFD)	• Lee, Shleifer, and Thaler (1991); Ross (2002); Chopra, Lee, Shleifer, and Thaler (1993); Uygur and Tas (2012). • Chen, Kan, and Miller (1993); Elton, Gruber, and Busse (1998); Ross (2005); Qiu and Welch (2006).	when individual investors become optimistic, smaller stocks do well. Transaction costs have greater influence than it was assumed at first. CEFD is a good proxy for investor sentiment. Criticism of the first CEFD study. The sample is biased and the return generating process is not much different from what would be expected by a chance. Rational explanation for the phenomena.
Proxies extracted from surveys consumer and investor confidence.	Otoo (1999); Brown and Cliff (2004, 2005); Fisher and Statman (2003); Lemmon and Portniaguina (2006); Schmeling (2009); Charoenrook (2005).	Changes in the consumer confidence may influence the stock indices. Mostly, these studies obtain a negative relation between mid–run up to long–run terms.
Indirect measures obtained from several market variables	Neal and Wheatley (1998); Wang, Keswani, and Taylor (2006); Baker and Wurgler (2006, 2007); Edelen, Marcus, and Tehranian (2010).	The investor sentiment has similar impact for value and growth stocks. However, large firms are likely to be less affected by the sentiment. Mostly, it is reported negative relation between investor sentiment and future stock returns.
Proxy extracted from public and social media	Clarke and Statman (1998); Fisher and Statman (2000); Tetlock (2007); Tetlock, Saar–Tsechansky, and Macskassy (2008).	A movement to more bullishness leads to lower conditional volatility and higher returns. The individual investors and newsletter writers are strongly influenced by past returns. Mostly, the results contain controversial evidence.
Proxy extracted from Internet message boards	Wysocki (1998, 1999); Tumarkin and Whitelaw (2001); Antweiler and Frank (2004); Fisher and Statman (2004); Das and Chen (2007).	No connection is revealed between message board activity and industry–adjusted returns or abnormal trading volume. Postings volume may positively correlate with volatility and bullishness. Mostly, the positive relation between sentiment index and stock index return on aggregate level is reported.

Source: Own work

One of the first and also the best known academic research of investor sentiment was made by **Lee et al (1991)**. They define the sentiment as biased expectation of the investors towards stock return that is unjustifiable by fundamental values and base their research on the market data of the *Closed–End Funds Discount* (*CEFD*). Leaning on the fact that retail investors are known to disproportionately hold closed–end funds, the *CEFD* indeed can be interpreted in the terms of investor sentiment. The researchers assume that subsequently to owning and trading closed–end funds in the US by only individual investors, the discount movements should reflect the differential sentiment of individual investors. They create a value–weighted index, based on the *CEFD* of 20 stock funds and observe high correlation among the funds, which may justify the construction of the value–weighted discount. They compare the changes in their value–weighted discount index with returns of portfolios of stocks with different market capitalization and conclude that when individual investors become optimistic, smaller stocks do well and retail investor sentiment moves oppositely to the discount increases. In addition, they emphasize that several factors have redemptions, like agency cost, are able to influence the *CEFD*. Ross (2002) explores these factors with further confirmation of such claim and argues that transaction costs have even greater influence than it was assumed by Lee et al (1991).

Chen et al (1993) provide a detailed critique of the Lee et al (1991) study. Based on 4 empirical issues[19], they believe that the results of the criticized studies are the consequence of its goal — to resolve the *close–end puzzle* and the *small firm effect* at the same time. The main argument is that the sample is biased and the using of the public utility during the regressions. Chopra et al (1993) respond to the critique, arguing that using of the public utility make it even more vivid that the stocks with similar relationship structure, but different fundamentals, move in the same direction. Responding to the biased sample argument, they run 3 additional regressions and still obtain similar results.

However, Elton et al (1998) introduce new critique of Lee et al (1991), arguing that sentiment index is involved in the return generating process more frequently than any other index, which could be combined in a similar manner. This means that change in the discount for closed–end funds enters the return generating process is not much different from what

[19] A role of utilities of small institutional ownership forms, the robustness of the relationship between fund discounts and the returns on high versus low ownership forms, the regression specification and the regression with only one closed–end fund.

would be expected by chance. In addition, using a more general multifactor model eliminates the patterns obtained in two–factor model of Lee et al (1991). Finally, they point that sentiment index is not empirically derived factor.

Ross (2005) suggests rational explanation for *closed–end fund puzzle*. He argues that agency costs, illiquidity of assets and tax liabilities affect the attractiveness of such funds, which leads to a disparity in trading price.

Closing the set of criticism, Qiu and Welch (2006) argue that the base for validation on the correlation of the *CEFD* with the small firm returns actually is driven by financial markets phenomena that are not yet fully understood. They provide significant evidence that *CEFD* alone may not be a sufficient proxy for all investor sentiment due to omitted variable problem or confounding variables. They fail to obtain any correlation between the *CEFD* and other proxies of investor sentiment. For this reason, the findings of Lee et al (1991) are considered as controversial. However, Uygur and Tas (2012) report confirming evidence to support that small firms outperform large firms when the *CEFD* decreases. With it they argue that the *CEFD* alone cannot be applied due to omitted variable problem or confounding variables supporting, Ross (2005) and Qiu and Welch (2006).

According to Kim and Kim (2012), in general, all the empirical behavioral literature about the sentiment, in accordance to its origin, can be divided into 4 groups. The origin is mostly prescribed by a specific sentiment proxy, used in a given empirical study, as follows:

- investor sentiment information from surveys consumer and investor confidence;
- indirect sentiment measures obtained from several market variables;
- investor sentiment proxies extracted from public and social media;
- information extracted from popular Internet message boards.

Sentiment measurement extracted from surveys of consumer and investor confidence

This group of tests contains studies based on proxies for investor sentiment which comes from surveys of consumer and investor confidence. Usually, the studies of this group, through the time–series tests, attempt to capture a relation between the investor sentiment indices and aggregate stock returns. Mostly, these studies obtain a negative relation between future stock returns and the investor sentiment in mid–run up to long–run terms.

One of the first studies in this group was performed by Otoo (1999). Using the ***Michigan Consumer Sentiment Index*** (***CSI***), which is nationally representative survey of about 500 US households and on the Wilshire 5000 stock index, he reports contemporaneously correlation between changes of equity values and changes in consumer sentiment. Within *Granger causality* test, he finds that increases in stock indices also cause consumer confidence to increase, but with a short lag. However, in the reverse direction it does not hold. Jansen and Nahuis (2003) extend the Otoo's (1999) analysis and find that the stock returns and changes in consumer confidence have positive correlation, confirming his results. In contrast, Christ and Bremmer (2003), by extending of Otoo's (1999) analysis to 3 common indices[20], are failed to obtain statistically significant correlation between the unexpected changes in consumer confidence and the stock prices. Since the consumer confidence is based on publicly available data, it should be already incorporated into stock prices, making a forecasting impossible, as the *EMH* suggests.

Based on *CSI* and on the ***Conference Broad's Consumer Confidence Index* (*CCI*)** Fisher and Statman (2003) argue that changes in the consumer confidence may influence the stock indices, i.e. the reverse direction also holds. In addition, they find a statistically significant relationship between changes in consumer confidence and changes in the sentiment of individual investors after construction of *AAII* sentiment index as the ratio of bullish investors to the sum of bullish and bearish investors. Charoenrook (2005) argues that his findings, from the behavioral finance view, reflect how investor sentiment is systematically correlated with stock prices and hence influences the aggregate stock market.

Lemmon and Portniaguina (2006), using both the *CSI* and *CCI* as measures for investor optimism, analyze the time–series relationship between sentiment and stock returns. They discover that investor sentiment has a negative effect on value stocks, but has no significant effect on growth stocks where particular *CSI* correlates especially well with small stocks and returns of firms held disproportionately by retail investors. This may indicate that investors are likely to overestimate small stocks relatively to large stocks during periods of high confidence. Though investor sentiment is able to forecast the returns of small stocks and returns of stocks with high levels of individual ownership, it fails to forecast time–series variation of *value* and *momentum* premiums. They document that consumer confidence is related to a various of macroeconomic variables, when *default spread* is the one most correlated with the *CSI*. It is obvious that investors are less fearful about the likelihood of a

[20] Dow Jones, the S&P500 and the NASDAQ.

potential crash during good economic states. Therefore, they change the jump risk premium incorporated in option prices. Schmeling (2009) focuses on the markets with low institutional development or markets which are especially tend to overreaction. He obtains negative relation between the sentiment and future expected returns across countries. He emphasizes that decreasing predictive power of sentiment as a horizon increases may indicate elimination of the *noise trading* effects in the long–run terms and strengthening of arbitrage, but in the short–run terms, there are limits to arbitrage as the behavioral theory argue. Also, he supports the two–way causality when sentiment depends on previous returns and the returns depend on previous sentiment. The findings of Schmeling (2009), which is consistent with the proves of Lemmon and Portniaguina (2006), are confirmed by Zouaoui, Nouyrigat, and Beer (2011).

Lee, Jiang, and Indro (2002) through the ***Investors' Intelligence*** **(*II*)** sentiment of independent advisory newsletters attempt to test the impact of *noise trader* risk on the formation of conditional volatility and expected returns. They assume that if the prediction of *DSSW* (1990) model is right, then empirical tests on the mean or variance of asset returns alone are misspecified and hence incomplete. They confirm negative effects on the market volatility by the changes in a sentiment though the same changes have positive effects on excess returns. Noise trading constantly influences the volatility that leads to updates in the expected return where the sentiment affects both: large/small–cap stocks.

Brown and Cliff (2004) construct *composite sentiment index* and argue that the sentiment should be formed through the process of time and some of the variables may reflect the same changes of sentiment sooner than others. Their results show that investor sentiment cannot predict future return in the short run and it is negatively correlated with returns of the next 1–3 years, but positively correlated with aggregated stock returns at the same time. Brown and Cliff (2005) examine the impact of the investor sentiment on deviations for the **intrinsic value** in the aggregate level of stock returns. This time they focus on a relation between excessive optimism and subsequent market overvaluation and also the relation between high sentiment and price reversing to its fundamentals. They conclude that investor sentiment has no ability to predict the returns on monthly and weekly basis, but based on the *bull–bear spread* from *II* survey, it may predict market returns over the next 1–3 years. The market is overvaluated during periods of high optimism and high current sentiment is followed by low cumulative long–run returns. In addition, the researchers observe that the investor sentiment influences growth stocks rather than value stocks. They demonstrate that sentiment may persistently and directly affect stock demand. They also demonstrate that the arbitrageurs adjust their sentiment downwards when they expect retail investor sentiment to be high, but the retail investors do not take into account the sentiment of the arbitrageurs, which supports the behavioral argument for limited arbitrage only in the long–run terms. The methodologies used in Brown and Cliff (2004, 2005) are widely acceptable in present days.

Sayim, Morris, and Rahman (2013) apply the *American Association of Individual Investor Index* to investigate the relation between the sentiment and the stocks of 5 US industries. They document a significant influence of the sentiment on stock return and volatility.

Indirect sentiment measures obtained from several market variables

This group of tests contains indirect sentiment measures extracted from observable market variables, similar to the approach of Brown and Cliff (2004). As well as within the previous group, it is reported negative relation between investor sentiment and future stock returns.

One of the earliest studies, Neal and Wheatley (1998), adopt 3 measures[21] of investor sentiment to predict future returns. It was obtained that the net mutual fund redemptions have an ability to forecast the size premium while the ratio of odd–lot sales and purchases is out of such ability. In general, the odd–lot ratio has very low predictive power while the other 2 factors have statistically significant explanatory ability.

Wang et al (2006), focusing on the relations between sentiment, returns and volatility, develop their unique sentiment measurement based on put/call ratio. According to their results, it is likely that returns and volatility cause the sentiment, but not the inverse. Hence, they conclude a limited predictive ability of sentiment in forecasting.

Baker and Wurgler (2006) extend the approach of Daniel and Titman (1997) and construct a *composite sentiment index*, similar to those of Brown and Cliff (2004), but this time extracted from 6 market variables where the number of *IPO*s with the lagged average first–day returns on *IPO*s, the 1 period lagged values of *NYSE* share turnover and the dividend premium represent high investor sentiment. While the *CEFD* represents the difference in price between the underlying asset and security price of the currently traded closed–end fund. In addition, they identify specific characteristics of firms that investor sentiment is more affected by, such as age, size or growth parameters. For their tests, Baker and Wurgler (2006) construct equally–

[21] The level of discount on closed–end funds, the ratio of odd–lot sales to purchases and net mutual fund redemptions.

weighted average returns for 16 spread portfolios which defined as *HML* in the sense of the Fama–French model, allowing positive or negative returns on these portfolios. Using the *PCA* to isolate the common sentiment component, they successfully obtain a cross–sectional effect of the sentiment when demand for certain stocks is sentiment–influenced and conclude that returns of stocks with highly subjective valuation and difficult to arbitrage are constrained by investor sentiment at the beginning of the period. The attractiveness of stock with high sentiment increases even more for the speculators but decreases for the arbitrageurs. For this reason, young, small, unprofitable, non–dividend payer, high volatility, extreme growth and distressed stocks — all tend to earn lower subsequent returns. They show that the investor sentiment has similar impact for both value and growth stocks and argue that due to their sentiment, investors construct a large spectrum of valuations in a case of the stocks of companies with short and instable earnings history and with apparently unlimited growth opportunities. However, large firms are likely to be less affected by the sentiment. The Baker–Wurgler *composite sentiment index* turned to the most widely adopted proxies in behavioral literature of present days. It has been used by authors, such as Mian and Sankaraguruswamy (2008), who obtain the relation between sentiment and price when investors tend to overreact to positive earnings surprises and by Chang, Faff, and Hwang (2009), who discover strong sentiment contagion since the US investor sentiment may reflect global sentiments.

Further, Baker and Wurgler (2007) examine companies with particularly sensitivity to investment sentiment such as young, growing, highly volatile, financially distressed, non–dividend payer firms with low capitalization and low profitability. In general, they demonstrate that investor sentiment contemporaneously positively correlates with aggregated stock returns, obtaining limited evidence for short–term predictability in returns. In contrast to the classical approach, they find that the higher risk stocks sometimes reflect lower returns meaning global and local sentiment have a predictive ability for the market returns as well as for relative returns of the young, growing, highly volatile, financially distressed, portfolios for 6 major stock markets. Baker, Wurgler, and Yuan (2012), extending the previous studies, provide international evidence for the significant explanatory power of investor sentiment, where past–year global sentiment may predict country–level returns for the following 12 months. The authors also confirm that investor sentiment affects aggregate market returns and has an influence on abnormal market returns subsequent to the market anomalies.

Stambaugh, Yu, and Yuan (2012), examining effect of investor sentiment on anomalies and defining the spread portfolio as the long/short portfolio such that the returns remain always positive, demonstrate that these portfolio returns are higher/lower subsequent to the high/low sentiment. Investor sentiment is a significant negative predictor for the short legs of long–short investment strategies. Additionally, they confirm the findings of Baker et al (2012).

Edelen et al (2010) suggest measuring the sentiment with differences in allocations between retail investors and institutional investors. Based on the Federal Reserve's quarterly Z.1 statistical release data, which provides quarterly holdings data for various categories of cash and cash equivalents, equities and fixed–income securities, they report negative relationship between investor sentiment and future stock returns. They demonstrate that when the share of retail investable wealth held in equity relative to the share of total investable wealth held in equity is high or low, subsequent stock market returns tend to be low or high respectively. They suggest that fluctuations in individual sentiment to those of institutional are the main non–fundamental driver of equity valuations.

Sentiment measurement extracted from public and social media

This group of studies is based on another type of indirect sentiment measure, which is extracted from available public and social information. Financial advisory newsletters, financial journals and other financial media are used to create a proxy for the sentiment. In general, here the researches contain controversial evidence about sentiment–return relation.

Clarke and Statman (1998) continue and extend the original work of Solt and Statman (1988), who focus on the nature of forecasts because forecast patterns affect returns, volatility and trading volume. They find no relation between the sentiment of investment newsletter writers and subsequent stock returns. Moreover, the investment suggestions of the writers affect the investment performance of individual investors in two ways:

- followers of newsletter writers pay money for newsletters whose advice is no better than a free toss of a coin[22];
- followers of newsletter writers move away from the optimal trade–off between risk and return in their strategic asset allocation in favor of faulty tactical asset allocation[23].

Using ***Bullish Sentiment Index*** **(*BSI*)** included in the *II* to measure sentiment of newsletter writers, the researchers fail to detect statistically significant relationship between the *Forecast*

[22] Original phrase from Clarke and Statman (1998).
[23] Original phrase from Clarke and Statman (1998).

Patterns Newsletter writers and the future direction of the stock market, because the patterns do not reflect true market information, but believes of the writers[24]. With it the newsletter writers sentiment is strongly affected by past returns and volatilities. The authors demonstrate how investor enthusiasm may correlate with the market:

- high short–run returns are associated with a move from bearishness to bullishness;
- high long–run returns are associated with 'nervous' bullishness.

A movement to more bullishness leads to lower conditional volatility and higher returns.

Fisher and Statman (2000) create groups of small or individual investors, medium or newsletter writers investors and large or institutional investors (Wall Street strategists) to measure sentiment of different classes of investors. They use weekly surveys from the ***American Association of Individual Investors*** **(*AAII*)** for the small investors group sentiment, the service of ***Investors Intelligence*** **(*II*)** to measure newsletter writers' sentiment and the data compiled by Merrill Lynch for measuring institutional sentiment. According to the results, the individual investors and newsletter writers are strongly influenced by past returns. In contrast to Lee et al (1991), the researchers find that individual investors and newsletter writers are much sensitive to the developments in *S&P500* rather than to the small–cap stocks. On the other hand, Wall Street strategists' opinions are not influenced by past returns. Individual investors do not act on their sentiment and hence their behavior is not completely irrational. The authors conclude that sentiment of individual and institutional investors are contrary indicators for future returns without statistically significant relation between the sentiment of newsletter writers and stock returns**.**

Tetlock (2007) pioneers to adopt the approach of *General Inquirer* textual analysis program through the *VAR model* in addition to the *Harvard IV–4–TagNeg* dictionary in order to identify negative meaning of a word. He concludes that news media content can play a role of a considerable proxy for investor sentiment. However, short–run return predictability quickly reverses at the market level and becomes weaker 2 days after releasing the relevant news completely disappearing within a week, supporting the idea of arbitrageurs. Tetlock et al (2008) extend the previous study. They are able to demonstrate that negative words have

[24] The tendency to identify patterns in random data is described by Gilovich, Vallone, and Tversky (1985), who argue that individuals expect to see random characteristics in large and small samples if the series supposed to be random, but when they find patterns in a small sample of the series, individuals reject the possibility that the series is really random.

statistically significant predictive power for quarterly accounting earnings and returns on the *S&P500* firms.

Sentiment measurement extracted from popular Internet message boards

This group of studies uses popular Internet message boards such as Yahoo!Finance and RagingBull.com as investor sentiment. To analyze the messages, the studies adopt distinct classifier machine learning algorithm. Traditionally, the positive relation between aggregate sentiment index and aggregate stock index return and level on the next trading day is reported. At the same time, significant relationship is found between sentiment and stock price changes on average across the individual stocks.

The first who paid attention that internet have a potential to provide a source for sentiment changes was Wysocki (1998, 1999). He studies posting activity on the web on 3,478 of the 8,011 firms from the Yahoo!Finance Message boards to investigate whether a number of messages may reflect next–day stock price changes. He finds that changes in overnight message posting volume of 50 firms with the highest activity on Yahoo! have a significant predictive power for next–day trading volume and abnormal returns. On the other hand, a daytime posting activity did not have any significant impact on the stock market activities. He documents that firms with high market value, return and accounting performance have the highest message posting activity and reports that the posting activity may strongly relate to various balance–sheet characteristics of a cross–section of companies. The firms with high short–seller activity, market values relative to fundamentals, low institutional holdings, high trading volume, extreme performance or extensive analyst following are more likely to generate high posting activity.

A very central work within the studies of sentiment through internet sources is done by Tumarkin and Whitelaw (2001) who examine the information embedded in voluntary user ratings (from strong buy to strong sell). Through a *one–day–lagged VAR–model* their study reveals no connection between message board activity and industry–adjusted returns or abnormal trading volume, which is more consistent with the *EMH*. With it they emphasize that buy and sell signals may carry very different information with respect to subsequent stock returns and the true information value of online messages becomes apparent only when measured against market–adjusted abnormal returns. However, this approach ignores much of the sample, because, for instance, only less than a quarter of all messages come with a user rating and evidence from stock message boards has shown that self–disclosed ratings are often biased. Tumarkin (2002) and Dewally (2003) continue/replicate the previous study and obtain similar conclusions.

Another central study within this class is conducted by Antweiler and Frank (2004), making qualitative data analysis in addition to quantitative data analysis. They decide to employ 2 popular classifiers to identify buy, sell or hold tone in the postings, distinguishing categories for stock messages, as bullish, bearish and neutral. Based on the counts of classified messages across time within the *Rainbow algorithm* of McCallum (1996), they constrain **bullishness index**. Using the *GARCH* method with linguistic content analysis, the authors find significant, but negative and contemporaneous correlation between the postings volume and the next–day stock returns, but still economically small relatively to transaction costs. However, stock postings cannot predict actual returns. They conclude that postings volume is positively correlated with volatility and bullishness, where the causality for volatility comes out from message boards to the market, but not in the reverse direction. With it, in their study, the sample period includes the burst of the internet bubble and dot.com companies with unsustainable business models and partly unrealistic valuations which represent a substantial share of the sample.

Fisher and Statman (2004) decide to investigate sentiment influence on the background of the 2000 millennium stock market bubble through the postings on Yahoo! message boards with results from the *Gallup/UBS* and *BusinessWeek* surveys. They report the same causality as in Fisher and Statman (2000, 2003). The authors show that Wall Street strategists are likely to be less bullish after large gains, but may become bullish after the market crash. At the same time, individual investors are likely to be bearish.

Another important study within this class is introduced by Das and Chen (2007) where the authors use a combination of several knowledge discovery algorithms applied to internet online boards such as Yahoo!. They propose two–step procedure/classification:

– During the 1st classification, the stock messages, based on analysis on posts from the ***Morgan Stanley High–Tech Index*** **(*MSH*)** message boards, through new natural language processing algorithm are labeled as having a "bearish", "bullish" or "neutral" sentiment. Within a voting mechanism of 5 classifiers, they label the postings with an *optimism* score, based on the in–text ratio of positive to negative terms from the ***Harvard Dictionary IV–4–TagNeg***.

- During the 2nd classification, additional filter is applied to classify the texts again on a basis of the reduced sample in order to remove ambiguous texts.

After the preprocessing procedure, they construct a sentiment index and apply it to 24 high–tech stocks. Since the researchers find that the sentiment has a weak predictive ability for future stock returns at the aggregate levels, but is out of such ability at the individual stock level, they conclude that the aggregation of sentiment reduces some of the *noise* from individual stock board messages.

Chen, De, Hu, and Hwang (2012, 2014) examine a potential influence of *peer–based advisory* on financial markets, extracting user–generated stock opinions from the most frequently visited personal finance social–media website, ***Seeking Alpha* (*SA*)**. Using similar technique of Das and Chen (2007), Tetlock (2007) and Tetlock et al (2008) combined with wording lists of Loughran and McDonald (2011), they report a negative relationship between sentiment and future stock returns, where the fraction of negative words or views negatively predicts firms stock returns. The relation is statistically significant and economically meaningful. Their study is closely related to previous Tumarkin and Whitelaw (2001), Antweiler and Frank (2004) and Das and Chen (2007) studies, though the results are different, which may be partly explained by a broader sample.

The empirical results from the sentiment literature are mixed, depending on the choice of a proxy for the investor sentiment. Mostly, it suggests 3 sentiment–return relationships with several exceptions:

- positive relation between changes in investor sentiment and stock returns, which may justify limits of arbitrage since the prices can be overvalued/undervalued during excessive optimism/pessimism contrary to the fundamentals;
- negative relationship between current investor sentiment and future stock returns which may indicate the price reversing to the fundamentals;
- causality and direction of the relation between investor sentiment and the stock returns is ambiguous.

Due to the complexity of measuring investor sentiment and the lack of a universally accepted proxies, measuring a sentiment can be a difficult task. This leads to a development of a wide range of methodologies and controversial empirical findings. A disagreement among the researchers what model should describe the financial reality the best also add to the disputability of the results for the tested models.

2.3. Tests of Technical Analysis

Technical analysis studies attempt to demonstrate a profitability of using trading systems. Some techniques are simple, like moving averages, but other can be complicate, involving computer power to proceed. The approach of technical analysis is disputable among the researchers since it suffers from lack of a testability. However, investors and fund managers still believe it can be useful in building an investing strategy. Indeed, despite the fact that question of profitability is opened, some technical tools can be helpful in financial analysis and in a decision–making process. They will be also added to the model introduced in Chapter 3 of this thesis. Here I will present the results obtained from the tests of technical analysis tools.

2.3.1. Historical context

Early empirical studies are focused on examination of the profitability of technical trading rules in various markets in order to reject the ***Random Walk Hypothesis*** **(*RWH*)** with followed up *Efficient Market Hypothesis* (*EMH*) and the results are significantly varying from market to market. Mostly, those studies emphasize limited evidence to support the profitability of technical trading rules. The technical trading rules are found to be applicable for futures markets and foreign exchange markets, while stock markets demonstrate more efficient behavior. The early studies are often criticized for their limitations in testing procedures such as data snooping, ignoring the riskiness of technical trading rules, avoiding statistical tests for significance and difficulty in the interpretation of the results.

The earliest study was performed by Stewart (1949), analyzing behavior of customers of a large Chicago futures commission firms. His results demonstrate that in general the technical trading is unsuccessful, but a representative successful speculator follows price trends.

In 1960, Donchian studies over 700 technical signals. He is the first to make comprehensive research paper. He suggests his own unique technique, ***The Donchian Channel***, for deducing long and short positions. Within the technique, he was able to obtain annual return up to 248% on the on margin of 1,000$. Lukac and Brorsen (1989) test 15 futures from agricultural, metals, currencies and interest rates. They adopt channel and directional movement, where for both systems 12 parameters and compare the results to the *B&H* strategy as a benchmark. They find that both technical signals generate statistically significant mean net returns over the benchmark. However, the trading systems yield similar profits across different optimization strategies and even different parameters. Thus, because fixed parameters captured the only consistent and statistically significant excess returns, the authors conclude that reoptimizing parameters offers no real value in capturing economic profit.

A study by Larsen (1960) focuses on the serial independence of price changes in Chicago corn futures prices. Based on Working (1958), he applies an *index of continuity* to compare a price change with subsequent changes over an interval without assuming that subsequent changes are dependent with any fixed time lag. In the study, both data sets demonstrate similar patterns. Analyzing the patterns, he highlights that 81% of price movements take place on the initial day, which followed by a 4–day period. His results demonstrate both positive and negative serial dependence of price movements.

Modern studies are characterized with more advanced testing methods. More statistical tools are applied to validate the results of trading rules, though the trading systems itself do not undergo much changes. The modern studies can be divided into several groups regarding to the testing procedures:

Standard studies use of parameter optimization, out–of–sample verification and statistical tests for trading profits. Mostly it indicates availability of technical trading to generate positive returns in speculative markets, but such results are under suspect of data snooping bias.

For example, Taylor (1992), adopting the revision price–trend model finds 3 technical trading systems and a revised statistical price–trend model, generate statistically significant excess mean net returns after transaction costs, which is real statistical prove against the random walk model and gives the explanation for forex prices movement. Silber (1994) concludes that after transaction costs his strategy is able to generate an annualized average return in excess of the benchmark for 9%–12%, emphasizing that since central banks operate in the currency and fixed–income markets to reduce exchange–rate volatility and manage inflation expectations, potential momentum profits are opened. Szakmary and Mathur (1997) conclude that central bank/governmental interventions change randomness, making markets directionally predictable. Further, Lee and Mathur (1996a, 1996b) are failed to obtain any statistically significant result for both in–sample and out–of–sample periods or the net returns are obtained negative. Lee, Gleason, and Mathur (2001), since trading rules are failed to capture statistically significant results across all currencies, conclude the predictive ability could not be reliable. Lee, Pan, and Liu (2001) applying the same trading rules obtain similar results. Maillet and Michel (2000) discover statistically significant profitability of the trading rules for all examined futures contracts, but the mark/franc that still able to generate positive excess return after retesting the results with the bootstrap approach.

Model–based bootstrap studies test popular technical trading rules in an effort to reduce data snooping problems. The studies indicate mixed results across markets and sample periods tested. Mostly, technical tradings can be profitable in emerging markets and foreign exchange markets. In contrast, they fail to demonstrate such profitability in developed markets. Unfortunately, these studies often ignore trading rule optimization and out–of–sample verification, which means that good profitability could be obtained by chance.

Marshall, Qian, and Young (2009) apply the bootstrap method and autoregressive process of order one *AR*(1), *GARCH–in–mean* and *E–GARCH*, suggesting to use market security characterizes such as market capitalization, turnover and volatility measured by standard deviation, for technical trading profits. Their results demonstrate very limited ability of the trading strategies to generate excess returns. Bajgrowicz and Scaillet (2012) through ***False Discovery Rate*** **(*FDR*)** to avoid data snooping bias, compare the results with the *B&H* strategy and with bootstrap reality check approach. They emphasize declining performance of the trading rules over time. Comparing to the bootstrap reality check approach, the *FDR* is able to detect such decline even earlier, though general conclusions from both approaches are very similar. The profits from the trading rules not only decline, but totally disappear during the last subperiod, which is a strong evidence to support Sullivan, Timmermann, and White (1999). Yu, Nartea, Gan, and Yao (2013) find that after applying transaction costs, the trading rules are no longer profitable more than the benchmark in all markets, supporting the weak form of the *EMH*.

White's *Reality Check* studies use of methodology to directly quantify the effects of data snooping. Despite the fact that *Reality Check* studies use a statistical procedure they also have some problems. For example, there is a difficulty in constructing the full universe of technical trading rules. Mostly, these studies indicate statistically significant profitability up to some period in which the profitability dramatically falls.

For example, Qi and Wu (2002, 2005)[25] obtain positive mean excess returns for moving average and channel breakout rules across all currencies after transaction costs considered. They discover that the results cannot be explained by systematic risk and seem to be robust to incorporation of transaction costs into the general trading model. Hsu and Kuan (2005)

[25] First appeared as working paper in 2002 and further converted to the article.

extend the study of Sullivan et al (1999). Their results demonstrate the superiority of a *Simple Moving Average* (*SMA*) over other simple strategies. As a strategy is complexed, the higher its profitability, however no strategy can consistently outperform the benchmark.

Chart patterns studies — here visible chart patterns generate technical signals. Generally, these studies demonstrate mixed results, depending on patterns used, markets and sample periods tested.

As shown in, Caginalp and Laurent (1998) who pay attention to the profitability of candlestick patterns and compare the results against average return, statistically significant predictability of short–term price changes. Leigh, Modani, Purvis, and Roberts (2002) and Leigh, Paz, and Purvis (2002) support such findings even after adjusting to the data snooping problems. Guillaume (2000) obtains statistically significant net returns for several trading rules during the 1st period, but not for the 2nd period, claiming that the head–and–shoulders patterns cannot be reliable indicator due to their instability over time. His findings are in contradict with Caginalp and Laurent (1998) conclusions. However, Lucke (2003) fails to generate positive mean returns for the chosen pattern after consideration of transaction costs. He also fails to obtain any correlation with central bank intervention, supporting the study of Guillaume (2000) and contradicting the studies of Leigh et al (2002). Through the kernel regression methodology, Dawson and Steeley (2003) apply the same technical patterns as in Lo, Mamaysky, and Wang (2000) and obtain similar results.

Wang and Chan (2009) propose absolutely novel approach to pattern recognition and create a **template grid** of rounding tops and saucers for buy signals detection. Their method is based on capturing reversal of price trend rather than on historical data. They conclude that the template may play a role of expert system, helping to the investors to make better decisions. Further, their mechanism is analyzed by Zapranis and Tsinaslanidis (2012). Without considering costs transactions, they obtain positive results for short–term horizons, concluding that such rule–based mechanism indeed can be seen as an expert system.

Nonlinear studies apply methods like the nearest neighbor or the feedforward network regressions. In general, these studies indicate availability of predictability and profitability abilities to the trading rules. In addition, non–linear studies suffer from the same problem as of genetic programming studies.

Fernández–Rodríguez, González–Martel, and Sosvilla–Rivero (2000) adopt a feedforward network model, comparing the results to the *B&H* benchmark. Their findings indicate superiority of the trading rules over the benchmark in terms of gross returns only for two subperiods. Sosvilla–Rivero, Andrada–Félix, and Fernández–Rodríguez (2002) adopt a trading rule based on the nearest neighbor regression. Their results at first glance are impressive however, after excluding days of the US intervention, those returns dramatically reduced, turning to negative and even far underperformed by the benchmark. Fernández–Rodríguez, Sosvilla–Rivero, and Andrada–Félix (2003) continue the line and discover the superiority of mean returns from trading rules, which generate statistically significant annual net returns.

Generally concluding and conducting the modern studies, it is possible to understand that trading systems can be profitable in the markets, connected to currencies, but not to equities where the results are mixed and can be harmed with data snooping bias. However, the profitability is likely to decline over time, such as in late 90s it becomes close to zero. The conclusion of the studies is that the markets have become more efficient with time, when the technological development contributes the most for the efficiency.

In contrast to previous modern studies, recent studies mostly discover uselessness of traditional technical trading based on price statistics, like moving averages or breakouts. They also discover that past results could be profitable during some specific period of time, but mostly such success is due to data snooping bias. Another area of recent studies is creating a multistage logarithm for an expert system. These studies demonstrate profitability of such systems and if not, the expert systems are still being useful tool during decision making process regarding to development of the investing strategy.

Summarizing the tests of the technical analysis through the historical timeline, it is possible to divide the results into early studies that indicate profitability of trading systems in different degrees and to modern studies that demonstrate weakening of such profitability to inability of profitability, arguing the *EMH* is valid. However, even in present days, the investors and financiers still use technical tools in their analysis, creating permanent noise on the markets.

2.3.2. Technical trading tools and strategies

Naturally, the trading tools and strategies are simple at the beginning, becoming more sophisticated with time. Due to development of computer technologies, the technical strategies became more complicated, however simple strategies have been never omitted. Most studies attempt to integrate and implement several technical techniques and strategies in the same research, making it absolutely difficult to refer every single technique. The examples of the studies have been presented above, further only representative and seminal studies for the *Technical Analysis* will be described. Those studies are pioneers within their classes with novelty of approach or methodology regarding popular trading systems as filter rules, moving averages, relative strength or momentum oscillators and integration between them, which have a strong connection to the variables introduced in the Chapter 3 and represented in Table 2.5.

Table 2.5 Technical indicators

INDICATOR	RESEARCH	CONCLUSSIONS
Filter rules	Alexander (1961, 1964); Fama and Blume (1966); Logue and Sweeney (1977); Peterson and Leuthold (1982); Sweeney (1986).	Mostly the studies demonstrate a possibility of positive returns on technical trading, rejecting the *EMH*. However, after applying transaction costs or dividend payoffs such profitability may fall dramatically.
Moving averages	Cootner (1962); Neftci and Policano (1984); Levich and Thomas (1993); Gençay (1998a, 1998b, 1999); Olson (2004).	Here the studies demonstrate mixed conclusions. Moving averages may lead to positive returns though its performance is not consistent or permanent. Winning in some markets, moving averages lose in the others. However, it is still be a useful analitycal tool.
Relative strength	Levy's (1967a,b); Bohan (1981); Pruitt and White (1988).	Securities with historically higher relative strength demonstrate on average higher return while securities with relative strength paired with volatility produce higher profits, though this without applying transaction costs, which reduce significantly these profits. Weekly rebalancing still able to generate positive profits within the relative strength basis.
Momentum oscillator, mixed trading and reality check	Smidt (1965b); Lukac and Brorsen (1990); Brock, Lakonishok, and LeBaron (*BLL* (1992)); Sullivan et al (1999).	Momentum oscillators can be useful in the long–run terms even after commissions and transaction costs. Other trading systems demonstrate statistical significance of portfolios returns, though with constant decline in the returns over time, concluding that trading rules cannot be sustainable. However, these findings probably are due to data snooping biases.

Source: Own work

Examined technical indicators: filter rules

Filter rules is a basic trading strategy generating buying/selling signals regarding to a given price barriers for up and down movements. One of the first and basic studies was performed by Alexander (1961) who applies his ***filter rules*** to identify nonlinear patterns in security prices. He creates a filter system from 1% to 50% comparing his results with *buy–and–hold* (*B&H*) strategy as a benchmark and obtains results in contradict to the *EMH*. Assuming that traders are able to buy at a price the subsequent low plus X% and sell at the subsequent high minus X%, he finds that small filter rules are able to generate larger gross profits than the *B&H* strategy and on average his rule is able to yield positive returns from 5%–30%. He finds a tendency of price changes to be followed by subsequent price changes in the same direction and concludes that short–term trends aside from the dominant long–term positive trend exists in stock market prices. Poole (1967) examines 9 exchange rates in international markets using filter of 10 rules comparing the results with *B&H* strategy within the assumption of Alexander (1961) and without applying transaction costs. The X% filters out the random magnitudes, allowing a speculator identifying the trend. If a real trend occurs, the filter analysis should indicate positive returns, otherwise it is zero. He obtains significant differences in filter and *B&H* strategies returns rejecting the *EMH*, though applying transaction costs may considerably pull his returns closer to the *B&H*. However, Mandelbrot (1963) argues that such approach is biased as the speculators are unable to switch from long (short) to short (long) at exactly the price if speculative prices follow a stable Paretian distribution function, but not Gaussian. Accommodating the criticism of Mandelbrot (1963), Alexander (1964) performs a new test to compare different moving averages and other technical signals. The results after applying the commissions demonstrate that only the largest filter outperforms the *B&H* strategy.

Fama and Blume (1966), in their influential work on technical trading rules, choose 30 individual securities from the *DJIA* and apply 24 filters, arguing that previous results of Alexander (1961, 1964) are biased, since dividends were not included into the data, which theoretically should reduce profitability of the filter rules. In addition, they argue that serial correlation is as powerful as the Alexander's (1961, 1964) filter rules for measuring the direction and degree of dependence in price changes. After including the dividend payments, only 2 securities have positive average returns and other show similar to *B&H* strategy results. After splitting the returns for long and short transactions, only 1 security has positive average returns on short and 13 securities have average returns per filter on long transactions. However, after breaking down trading positions into long and short, some filters are able to generate higher average returns on long positions. Despite these findings, Fama and Blume (1966) argue that profitability of long transactions is not better than *B&H* strategy after taking into consideration the commissions and operating expenses of the filter rules. Hence, they conclude that there is no potential in practical usage of the filter techniques. Sweeney (1988) reexamines the study of Fama and Blume (1966) and obtains significant positive returns for past winners even after transaction costs adjusted. Portfolio returns appeared to be robust across several subsamples with some sensitivity to transaction costs. He concludes that such good performance is due to applying his filter rule to individual security, but not to the market index as in Fama and Blume (1966).

Martell and Philippatos (1974) centralize on a question of whether the martingale hypothesis holds for September wheat and soybean futures contracts. They apply 2 learning models with a combination of adaptive filters rules for tests of efficient markets. Their estimation is done within an entropy format, using the log of probabilities of a forecast occurrence, which is expected to obtain zero return compared to the *B&H* strategy for martingale processes. However, their findings demonstrate the opposite, since returns are higher and less risky compared to the *B&H* strategy, but only for wheat futures. Though variance in net profits was consistently smaller compared to the *B&H* strategy in both markets, they claim that the validity of the martingale model is rejected[26]. The Martell–Philippatos model is consistent with DeBondt and Thaler (1985) overreaction process, because the conditions set for adaptive modeling are satisfied.

Logue and Sweeney (1977) examine the impact of transaction and interest costs on gross profit. They focus only on the French franc–US dollar exchange rate, comparing the results to a *B&H* strategy. Locational arbitrage occurs when two currencies and two markets are involved while triangular arbitrage occurs where three markets and three currencies are involved. Hence, triangular arbitrage is consistent with double converting of one currency to another with third currency involved, while locational arbitrage is the exchange rates for the same currency at different exchange outlets. Appling 14 filter rules and using spectral analysis they discover that 13 filter rules outperform the *B&H* strategy after considering transaction costs over a 4–year period. Applying their strategy to the French government securities they discover that 10 rules generate higher profits.

Peterson and Leuthold (1982) focus on 7 hog futures contracts from *CME*, applying 20 filter rules and compare the results to zero–mean profit benchmark. They discover that larger filters generate larger mean profits with larger variance even after commission charges. In general,

[26] Martell (1976) continues the previous study and introduces new adaptive model for the same sample. Though the new model outperforms the previous, it demonstrates unstability with respect to the information constraint. Also, since the model does not allow reflecting new information in a given period for a filter size, it can be better than pure information model.

all rules are able to generate mean gross profits, increasing with larger filters.

Cornell and Dietrich (1978) apply 13 rules of 0.1%–5% and moving averages with 0.1%–2% bands to 6 spot foreign currencies. They suggest to use *S&P500* for world portfolio proxies and generally calculating the *beta* of foreign currencies with the index rather than the *beta* of currency positions that results from technical trading rules. Their results exhibit some positive evidence on filter and moving averages rules of higher profitability even after applying of transactions costs and interest rate carry. Their estimates of the *beta* suggest that high returns of the 3 currencies cannot be a compensation for bearing systematic risk. Instead, low *beta* indicates that the investing in foreign currency provides a good hedge for an investor, whose portfolio is primarily centered on the US stocks. Further, Taylor (1992) uses such technique and obtains similar results.

Another influential study which stands in contrast with the study of Cornell and Dietrich (1978) was proposed by Sweeney (1986). His approach is much different while he is the first to systematically integrate risk–adjustment into the empirical examination of long position rules applying 7 rules to 9 currencies. Based on the assumption that the *CAPM* is able to explain excess returns of filter rules and of *B&H* strategy with risk premia are constant over time, he constructs an X–statistic to define the difference between filter and *B&H* strategy returns with transaction costs, post–sample performance and statistical tests considered. His results demonstrate that almost all filters outperform the benchmark. In addition, he tests 10 foreign currencies and finds that the filter rules statistically significantly outperform the benchmark. In his additional test for statistical significance of excess returns, he proves the technical trading is due to alpha generating ability of the signal strategy. The assumption that the *CAPM* should explain returns to both *B&H* and filters strategies implies that expected excess returns should not be significantly different from zero. However, the results of the study demonstrate the opposite, suggesting that the *CAPM* fails to explain price behavior in foreign exchange markets. Sweeney (1986) concludes that the *EMH* fails, but efficient markets hypothesis may hold if risk premia vary over time. In this case, positive returns on the filter rules may reflect higher average risk borne, but not true profits.

Examined technical indicators: moving averages

One of the earliest studies involving moving average was introduced by Cootner (1962) who uses moving average of a 5% band, comparing his results with *B&H* strategy. Moving average is designed to identify long– and short–term trends. He finds that though net returns from moving average rules are not much different from those of the *B&H* strategy, long transactions are able to generate higher returns. The variance of the trading rule is more stable. He concludes that the stock market does not follow a random walk. However, Van Horne and Parker (1967) conclude that *B&H* strategy outperforms all banded moving averages. Later, Van Horne and Parker (1968) compare simple as well as exponential moving averages and find that no one of them can outperform the benchmark permanently at any band. The same conclusion is reached by James (1968).

Dale and Workman (1980) apply 11 rules of moving averages to the US T–bill futures market without regarding to any benchmark. They claim positive average net returns, though the rules do not demonstrate consistent performances over the sample period. Large variance and the fact that other 10 rules are failed to obtain positive average returns, push them to conclude the results are rather due to luck.

Solt and Swanson (1981) analyze the data of London Gold Market with the data of silver from Handy&Harman. They create a combination of filter rules with moving averages, which they believe is much more accurate a buy/sell signal. Chosen benchmark is the *B&H* strategy. Their results indicate that only one filter rule has a better performance than the benchmark after consideration of the transaction costs. Other filter rules as well as moving averages are failed to exhibit any superiority over *B&H* strategy. Their results seem to indicate that combining signals does not add a significant value.

Neftci and Policano (1984) turn to futures market for applying moving averages along with a unique trendline or slope method without any benchmark considered. The slope method is designed to search for timing opportunities based on the security's price mean reverting back to the trendline after a significant enough deviation has occurred. The buy/sell signals obtained from the moving averages and the slope method are transformed into dummy variables and further incorporated into *OLS* linear regression model. Through *F*–tests, the authors obtain statistically significant coefficients for the dummy variable. The regression is able to produce consistent positive mean profits from gold, soybean and T–bill futures contracts, but fails to do so for copper futures.

Sweeney and Surajaras (1989) test filter rules, single and double simple moving averages strategies of currency futures contracts, using an equally–weighted and a variably–weighted portfolios over a 6–year period. They find that most trading systems are able to generate risk–

adjusted mean net profits after transaction costs. In general, the variably–weighted portfolio demonstrates a better performance than the equally–weighted portfolio however rebalancing optimization technique is not likely to improve either profits or risk efficiency.

Levich and Thomas (1993) introduce some novel method to measure the statistical significance of the moving averages and filter rule returns. Their data contains daily observations from 5 Chicago IMM currency futures. They apply a bootstrap approach to the raw returns on the contracts. To determine the profit generating process is not merely to chance, the authors create random series by permutation of the actual series of exchange rate changes with repeating this process 10,000 times and comparing to the profits of the randomly generated series. Such approach allows them generating an empirical distribution of profits. However, the authors emphasize existence of possible data snooping and suggest avoiding the problem by applying more moving average lengths along with other technical models. The results of the study demonstrate profitability of trading rules over the *B&H* benchmark, though transaction costs are able to reduce the profitability of some rules. The moving average rules demonstrate a better performance than the filter rules. They find that across trading rules from both trading systems, average profits of all currencies, except the Canadian dollar, are substantial. With it the profits are likely to decline in their final subsample.

Taylor (1994) examines 4 currency contracts futures and analyzes several price channels, comparing his results with zero–mean profits benchmark. For price series generated by ***Autoregressive Integrated Moving Average*** **(*ARIMA*)**(1,1,1) model, optimal price channel rules correctly predict sign of conditional expected returns of each currency futures contract around 60% probability in the out–of–sample period. He obtains 6.9% annualized average rate of return even after transaction costs. The results are significant at the 2.5% level. This study provides a proof for forecasting ability of price movements by the *Technical Analysis*. Thus, it is a proof that prices walk non–randomly through the time, as assumed by the *EMH*.

Gençay (1998a, 1998b) adopts trading system based on a feedforward network model which is artificial neural networks. This method develops buy/sell signals regarding to a function of past returns. He applies daily observations from *DJIA*, comparing the results to *B&H* benchmark or *OLS* model with lagged returns as regressors. Gençay (1998a) discovers that net returns of trading rules outperform the *B&H* benchmark, where Sharpe ratio tests support the results. Gençay (1998b) shows superiority for prediction ability of the technical trading of at least 10% over the benchmark. The rule with shorter moving average demonstrates even better out–of–sample predictions contrary to longer moving average. Gençay and Stengos (1998) continuing the previous studying line, obtain similar results. The *OLS* and *GARCH–in–mean* (1,1) are not able to outperform feedforward network models. Gençay (1999) continues to examine trading rules based on a feedforward network model and on the nearest neighbor regression in addition, comparing the results to random walk and *GARCH* (1,1) models. He discovers that non–linear models outperform the benchmark and that models with integrated 1/50 moving average has higher forecasting ability than those of 1/200 moving average.

Olson (2004) trades daily 18 currency exchange employing short moving averages of 1–12 days aside to long moving averages of 5–200 days, comparing the results to the *B&H* strategy. He discovers that the trading systems reduce its power to generate excess returns over time. He obtains about 3% of excess returns for all currencies, during 80s, but during 90s the excess returns for all currencies approached to zero, indicating increased market efficiency. Lo (2004) within the example of Olson (2004) emphasizes that such occurrence is normal, since the markets change its structure and so the market efficiency. The market efficiency is a dynamic process and hence, during some periods technical trading are more successful while it fails in other periods. Markets change also subsequently to the exogenous variables like central banks or governmental interventions. His theory formulated in the terms of ***Adaptive Market Hypothesis*** **(*AMH*)**.

Examined technical indicators: relative strength

One of the first studies for investigation rules of ***relative strength*** or ***portfolio updating*** was introduced by Levy (1967a,b). It is based on creation of ratios of current price to its average price over previous 27 weeks and ranking the results with further investing in high–ranked ratio portfolios, assuming that stocks move randomly and when a security diverges significantly from its 26–week simple moving average, the stock price would revert back to the trendline. Levy (1967a,b) performs a number of empirical tests with further ambitious final conclusion that random walk theory is refuted. Securities with historically higher relative strength demonstrate on average higher return while securities with relative strength paired with volatility produce higher profits. Jensen (1967), commenting on Levy's (1967a,b) findings, argues the results do not support such conclusions due to errors of definition of naive standard of comparison as the geometric average, definition and treatment of the risks of the random selection policy and the trading rules, problems associated with sampling error or selection bias and implicit assumption that all trades can be executed at the same prices. Jensen and Benington (1970), continuing investigation of relative strength, find that after applying transaction costs, Levy's (1967a) trading rules lose its superiority and the risk–adjusted returns are much lower than those of *B&H* returns. Akemann and Keller (1977), investigating relative strength indicator after applying transaction costs, obtain higher returns relatively to the benchmark, but the variance obtained is also much higher. They cannot conclude that relative strength strategy is absolutely preferred over the *B&H* strategy in terms of pure skill. In contrast to previous studies, Arnott (1979) suggests a linear regression model to analyze *beta* modified relative strength without regarding to any benchmark. Investigating the correlations between future returns and his own indicator of choice, he demonstrates that for the base periods of 1–18 weeks, the correlation between the change in beta–adjusted relative strength during a base period and during subsequent period is strongly negative. He states that careless use of relative strength can be a reason for serious money losses.

Bohan (1981), examining relative strength, constructs a strategy of buying the highest relative strength quintile against selling the lowest relative strength quintile and compares the results to simple *B&H* strategy. He discovers a strong correlation between the performance of the strongest and weakest industry groups in 1 year and that of the following years. In other groups was not detected much predictive significance. From here, the strategy with weekly rebalancing allows obtaining of consistently positive excess returns. Brush and Boles (1983) continue and extend the study of Bohan (1981). They form equal weighted deciles by buying the top decile and selling the bottom decile with monthly rebalancing, comparing the results to *S&P500* index, including dividend payments. The Brush–Boles model demonstrates compounded growth of 15.2% per year against 5.9% for the *S&P500* index even after transaction costs and dividends incorporated. Their model pulls a lot of attention of hedge funds and portfolio managers. Brush (1986) continues previous study and the study of Bohan (1981), comparing performance of 8 different relative strength models to equal–weight version of the *S&P500* index for the benchmark. His most successful model obtains 5% excess return after avoiding the year–end effect and exploiting *beta* corrections with the negative predictive power of 1–month trends, additionally to the transaction costs. He emphasizes that *value* and relative strength are likely to generate returns that offset each other. From here, a portfolio that combines both strategies should generate higher returns compared to those of each strategy separately.

Pruitt and White (1988) apply a unique combined trading system, which is the ***Cumulative volume, Relative Strength and Moving Average*** **(*CRISMA*)**. According to their approach a buy signal is received when a stock complies with 3 conditions:

(1) the 50–day simple moving average must cross the 200–day simple moving average from below at a time when the slope of the 200–day simple moving average is either zero or positive;

(2) the relative strength line must have a slope equal to or greater than zero for all points over the previous 4 weeks;

(3) the cumulative volume graph must have a positive slope from its starting to its ending point over the preceding 4 weeks.

The point at which a stock's 50–day *SMA* crosses its 200–day *SMA* from below establishes the stock's base price. Further, a filter of 10% is applied, buying a stock with 110% of its base price. When the stock's price falls below its 200–day *SMA* or it increases 120% above its base price, the sell signal occurs. Pruitt and White (1988) use Scholes and Williams (1977) market model, the *OLS* market model, the market–adjusted returns model and the mean–adjusted returns model for the expected returns estimation and compare the results to *B&H* strategy. The results demonstrate superiority of *CRISMA* strategy over the benchmark even after adjustments for risk and transaction costs.

Studies of momentum oscillator, mixed trading and reality check

Smidt (1965b) pioneers and proposes a technical trading of ***momentum oscillator*** of 40 rules, indicating whether a security is overbought/oversold by analyzing recent gains/losses within a specific timeframe. He examines the reaction of speculators to a new information on May soybean futures market and argues that if all traders have the same level of informativity, there are little possibilities to gain extra profit. In contrast, if some part of the traders is not informed at the same level, there is a systematic tendency for a price rise (fall) to be followed by a subsequent further rise (fall). In this case the technical trading can be useful in the long–run terms. He demonstrates that 70% of all *momentum oscillators* after commissions lead to significant positive returns. His technical signals are useful on the agricultural commodity and forex markets, but completely fail to do so on the equity markets.

Irwin and Uhrig (1984) suggest a study on 8 commodity futures contracts. They apply

Donchian inspired channels, simple moving averages and momentum oscillators for 3 in–sample and out–of–sample periods, comparing the results to a zero–mean profit benchmark with commissions implied. The authors discover that all 3 types of technical signals are able to capture positive mean profits for all contracts.

Chang and Olser (1999) introduce a study on 6 daily currency futures. They apply head–and–shoulders pattern, moving averages and momentum lags, comparing the results to the simple *B&H* strategy. They discover that head–and–shoulders pattern demonstrates profitability for mark and yen, but not for other futures. The momentum oscillators and simple moving average rules generate positive statistically significant mean excess returns for all currency futures, after taking transaction costs into consideration.

Neely, Papach, Tu, and **Zhou (2014)** perform a comparison of predictive ability between 14 most common technical indicators, such as moving averages, on–balance volume and momentum oscillators and between 14 popular macroeconomic variables, such as financial ratios. Further, through the *PCA*, the influence of every type of the variables on the equity premium has been figured out. The overall period is divided to in–sample and out–of–sample periods with *S&P500* monthly data involved in the estimation. They detect predictive ability for both: technical indicators and macroeconomic variables, whereas technical indicators demonstrate higher ability than those of macroeconomic variables. Moreover, integrated regression demonstrates that technical analysis is a perfectly complemental component in explaining the equity premium in addition to the financial ratios. The authors argue that the reason is the lack of market efficiency and provide 4 theoretical explanations:

(1) timing for receiving new information;
(2) heterogeneous investors;
(3) under/overreaction to new information;
(4) influence of investor sentiment.

Lukac and Brorsen (1990) choose 30 futures from 6 markets and apply 23 trading systems[27]. The results are compared to zero–mean profit benchmark with 100$ round trip transaction costs. Relying on the fact that historical volatility persists in the short terms while reverting to the mean in the long terms, they are the first to argue that the utility of volatility may play a role for technical trading signal. They discover that 20 trading systems generate positive mean

[27] Channels, moving averages, momentum oscillators, trailing stops, point and figure charts, a counter–trend model, a volatility based–model and combinations of different systems.

profits while 3 others generate negative mean profits. The researchers conclude that the short–term disequilibrium model is more appropriate than the random walk model in describing futures prices daily behavior, meaning that technical trading returns are positively skewed and leptokurtic. Hence, they argue that past applications of *t*–tests to technical trading returns in the studies by James (1968) and Peterson and Leuthold (1982) might be biased.

An important milestone in the field of *Technical Analysis* is the seminal work of **Brock, Lakonishok,** and **LeBaron (*BLL* (1992))**. They are first to change the academic view on *Technical Analysis* as it was largely dismissed by academics in the 1960s and 1970s. They perform their test within the daily data of the *DJIA* index during 1897–1986 period. In addition, the sample period was divided to 4 subperiods, which were chosen regarding to a significant influential event, like *WWII* or *Great Depression*, to obtain extra results. They adopt several simple moving average rules in addition to trading–range breakout, comparing the results with a benchmark of holding cash money. When a buy/sell signal occurs, ***Fixed–length Moving Average*** **(*FMA*)** for the ten days following a crossover and ***Variable Moving Average*** **(*VMA*)** records index returns until another crossover signal is occurred strategies are applied. For the support/resistance rules, buy/sell signal occurs when index price surpasses/declines below a pre–determined local maximum/minimum price. *BLL* (1992) compute 3 local maximums and minimums over the previous 50, 150 and 200 days with 3 additional rules computation by adding a 1% filter to the first 3 rules. The chosen holding period is 10 days, following buy/sell signals. Twenty moving average rules and 6 support/resistance rules include *BLL*'s 26 trading strategies are compared and analyzed in their work through *GARCH*–type estimations.

Lukac and Brorsen (1990) already emphasized that standard *t*–tests for the statistical significance can be problematic because of normal, stationary and time–independent distributions assumptions, which is not legitimate since one or more of these assumptions is very often violated in asset returns. In turn, *BLL* (1992) suggest to overcome such problem with ***bootstrapping technique*** approach of Efron (1979), which has been recognized today as the established one and further used in other studies, like Mills (1997). *BLL* (1992) discover that all of 26 strategies significantly outperform the benchmark. They discover that buy signals produce positive returns while sell signals produce negative returns that indicate inversely forecasting ability. The buy returns are even less volatile than the sell returns. Their findings are especially strong since every single trading rule they consider generates excess returns over the benchmark. Their results are also found to be robust, which means that the *EMH* does not hold even in its weak form.

Applying *BLL* (1992) methodology, next examples can be found. Bessembinder and Chan (1995) demonstrate that US–based signals can be successfully applied to Asian stock index markets. Bessembinder and Chan (1998) evaluate statistical significance of portfolios returns individual trading rules with constant decline in the returns over time, concluding that trading rules cannot be sustainable. Kho (1996) concentrating on 4 currency futures discover that buy signals are able to generate around 10% of annualized mean returns; but comparing the results to conditional form of the *CAPM*, it was found statistically insignificant. Raj and Thurston (1996) are able to generate statistically significant returns from buy signals of all moving average rules. Hudson, Dempsey, and Keasey (1996), Coutts and Cheung (2000) and Parisi and Vasquez (2000) discover positive statistically significant excess returns from the buy signals and negative statistically significant excess returns from the sell signals as in *BLL* (1992). Mills (1997) finds that trading rules are able to generate excess returns over the *B&H* strategy, though the results are statistically insignificant, especially for later subperiods. The same is about trading range breakout rules and *AR–ARCH* bootstraps approach. LeBaron (1999) obtains statistically significant results for the trading rules, even after adjustment to transaction costs, concluding the trading rules can be profitable during the periods of Federal Reserve interventions. Neely (2002) discovers that moving average rules are able to generate positive annual mean returns for all series however a central bank's interventions are not likely to influence the technical trading profits. Sapp (2004) comparing his results against Sharp ratio of *S&P500* index, obtains positive statistically significant returns until 1995, but further, those returns being positive turn to insignificant. Taylor (2014) applies the moving averages and breakouts as in *BLL* (1992) to daily data from the *DJIA* during 1928–2012. This sample is chosen for its historical importance as a measure of overall stock market performance and long time period record. He discovers within econometric models for risk–adjusted returns that profits evolve slowly over time. His findings are consistent with the *AMH*. In addition, he discovers undocumented phenomenon that leads to success of trading rules and this is ability of short–sell stocks.

One more seminal study of *Technical Analysis* was made by **Sullivan et al (1999)**. They reexamine the results of *BLL* (1992), demonstrating the ascent of the same significant profitability within the *DJIA* or the *S&P500* futures data. Further they conclude that the trading rules performance declines over time because the markets have become more efficient, eliminating profit opportunities. Through White's ***Bootstrap Reality Check*** methodology they apply 8,000 trading rules. The performance is measured with the mean returns and the Sharpe ratio, compared to the mean return criterion of the "null" system or to the risk–free respectively. According to their results, the best trading rule was 5–day moving average, indicating 17.2% of annual mean return, though the same rule indicates only 2.8% for out–of–sample period. Such rule has low statistical significance, indicating declining ability to generate valuable economic signals in the subsequent period. The trading system of *BLL* (1992) is concluded to be robust to data snooping biases and applying the Sharp ratio reveals statistical insignificance during several subperiods. For the *S&P500* futures the best rule generates a mean return of 9.4%, which is also concluded is the result of data snooping biases. Further, Sullivan et al (2003) propose another study based on the calendar effects trading system in addition to the rules from their previous study. Through the same *Reality Check Test*, most of the system revealed to be insignificant. The authors conclude that trading rules cannot hold consistent forecasting abilities, supporting market efficiency.

Hsu, Hsu, and Kuan (2010) are to test 16,500 trading rules on growth, developing markets and on *Exchange Traded Funds* (*ETF*). The sample period contains pre–*ETF* and post–*ETF*. To deal with the data snooping bias, the authors apply a novel **Stepwise SPA test**, which combines regular SPA test with StepM test. They provide a proof and simulation, demonstrating that their method has even better results than *Reality Check* or than SPA test and StepM test separately. Their results also show that trading rules have predictive ability in pre–*ETF* while absolutely lose it during post–*ETF*, without supporting with statistical significance. Kuang, Schröder, and Wang (2014) apply 25,988 trading rules consist with patterns used by Lo et al (2000). They also apply *Reality Check* method, *SPA* test, *StepM* test and *Stepwise SPA* test for controlling data snooping bias. The authors discover that hundreds or thousands of trading rules for every currency according to both mean excess return and Sharpe ratio criteria are profitable. However, after considering the data snooping bias, none of those rules profitable anymore. With this study, the authors prove that almost all profits from technical trading are probably due to data mining bias and point out the efficiency in Forex markets.

Despite the fact that the technical trading weakened with the time, it can still useful in our days. Its profitability can hardly be defined as impressive, keeping a place to find a better solution by possible integration of different approaches and theories into one universal platform. In Chapter 3 of this thesis, I introduce a suggestion for a new capital asset pricing model with full description and tests, which combine normative and behavioral aspects as well as those of technical analysis.

Chapter 3

Methodology and tests of unified capital asset pricing model

3.1. Fundamentals of the model

In 1965 Fama in his thesis formulates one of the most important financial principals, which is the *Efficient Market Hypothesis* (*EMH*). According to the *EMH*, an investor should be compensated for bearing the risk as much as it is allowed by the market in terms of its volatility. The prices are unpredictable due to a random walk and thus no abnormal profit is allowed, but exceptionally due to a chance. No past information can be useful to forecast future returns. That is because any available and new information is already incorporated in the securities' prices and the process is rapid. The investors are rational and if any irrational activity is detected on the markets, the investors are able to recognize it immediately and to act against such activity reversing the prices. The market is a kind of a mechanism, which leads to a situation where at any given point of time the prices reflect an intrinsic value of a firm. All this implies that market actual price (***P***) is always the fundamental price (***F***): $\boldsymbol{P} = \boldsymbol{F}$. As Fama (1965) believes, the actual market price is the best approximation for the fundamental price, since the intrinsic value is not always known.

The picture begins to change since Shiller (1981) introduces his a hundred–years graph, demonstrating that implied discounted dividend payments lead to high volatile actual prices than it should be as ex–post rational prices according to the *EMH*. The *EMH* demonstrates smooth line while discounted dividend line demonstrates high spikes, with opposite to the *EMH* price movements in some cases. This implies that in addition to the fundamental price exists non–fundamental (***NF***) component, which means that the actual price is not equal to the fundamental one: $\boldsymbol{P} \neq \boldsymbol{F} \rightarrow \boldsymbol{P} = \boldsymbol{F} + \boldsymbol{NF}$. Shiller (1981) tends to explain the deviation from fundamental price by the investors' heterogeneity that comes out from personal psychological abilities.

One of the earliest explanations for the phenomenon is provided by Black (1986). From the rational point of view, the prices should be fundamental and if they are not, there is something that prevents to reach it. Black (1986) defines it as *noise*. The noise is inability of rational investors to reach fundamental prices due to wrong information or late respond to actual information. He points economic advisory and *technical analysis* are among sources of such wrong information. The investors, aimed with wrong information, falsely think that it can be helpful to make a better decision and still act as the *EMH* investors, creating the noise. From here, if the non–fundamental component is noise (***N***) than this implies that: $\boldsymbol{P} = \boldsymbol{F} + \boldsymbol{N}$.

Started with Kahenman and Tversky (1979) and Shiller (1981) the noise is explained in behavioral terms. The investors cannot be rational in the sense of the *EMH* due to psychological biases. Personal abilities lead to different understanding of the same subject by different investors and hence to different reaction on new actual information. Three most common biases are generalized by Szyszka (2009) into a behavioral component (***B***). Within his model, Szyszka (2009) summarizes a large block of typical behavioral models, explaining the deviation from the fundamental prices by psychological factors. The model of Szyszka (2009) directly implies that: $\boldsymbol{P} = \boldsymbol{F} + \boldsymbol{B}$.

From the literature, it is possible to understand that noise is either rational, according to the normativists, or behavioral, according to the behaviorists: $\boldsymbol{NF} = \boldsymbol{N} = \boldsymbol{B}$. The only parameter which defines a segment of a price formation, as agreed by both the normativists and the behaviorists, is the fundamental component. Majority of existing models describes a universe of two groups of traders; smart traders in the sense of the *EMH* and noise traders (for example, *BSV* (1998), *DHS* (1998) or Szyszka (2009)). However, the universe should contain **all** possible investors, who may affect the market price and to create additional systematic risk. I suggest three groups for universe of the traders as follows: $\boldsymbol{P} = \boldsymbol{F} + (\boldsymbol{N} + \boldsymbol{B})$.

At any given point of time, an individual is free to decide to which group to belong to and to switch between the groups. However, in a given point of time the same individual makes his decision according only one preferred basis within 3 possibilities allowed:

– ***Absolute rationality*** — these investors act in the sense of the *EMH*. They are very smart with full access to the necessary information. They have rational expectations and thus this type of investors conducts the fundamental component (F_t) of a price formation.
– ***Rational non–rationality*** — these investors also act in the sense of the *EMH*, though this time they make their decision accordingly to the technical analysis, creating the noise (N_t), which is not rational from the point of view of fully rational investors.
– ***Behavioral non–rationality*** — these investors act in the sense of Szyszka (2009). The investors have psychological biases, like overconfidence or self–attributing, which are expressed by generalized behavioral component (B_t). However, in contrast to Szyszka (2009), here the behavioral component generalizes **all** possible relevant psychological

biases, creating overall market sentiment.

Hence, in a given point of time all the individuals are divided between only 3 groups and there is no possibility to any one of them to make the decision on several bases. From above, at any given point of time, 3 powers influence the price formations:

$$\boldsymbol{P_t} = \boldsymbol{F_t} + (\boldsymbol{N_t} + \boldsymbol{B_t}). \tag{3.1}$$

All the powers are independent from each other while in some cases they may even offset each other. For example, in a case where $N_t = -B_t$, the market price equals to a fundamental one. Nevertheless, if the noise and behavioral components are both positive, the deviation from the fundamental is much higher. Any other combinations between all 3 components may explain several market anomalies like over/under pricing or bubbles.

3.2. Methodology of the test

The main goal of this study is **to create a Unified Asset Pricing Model and to verify whether the nature of the aberrant from the fundamental price (*F*) is both — rational (*N*) and behavioral (*B*). This means that the Unified Asset Pricing Model should explain more appropriate the stock returns behavior than traditional or behavioral models separately.**

The study includes 4 stages as described in Scheme 3.1. During the 1st stage all necessary variables for the research are defined. There are 3 categories of independent variables: fundamental R_t^F for rational investors; noise R_t^N for rational investors who turns to technical analysis as well as economic advisory and behavioral R_t^B for investors who has a psychological bias. Within this stage goals and hypothesis are defined. The excess stock return ($RIRF_t^i$) is the dependent variable.

The variables used in tests of the Unified Capital Asset Pricing Model are popular and known in the related literature. The analysis procedure follows some existing studies. However, not all the original methodology of those studies is implemented here. Some variables are modified in order to add novelty and in parallel to validate the original methodologies.

The 2nd stage of the study introduces the results of models, derived from the *Principal Component Analysis* (*PCA*). Three models are created where one stands for the technical analysis; one stands for the sentiment and one model stands for the unified component, where the technical and sentiment indicators are integrated together through the *PCA*. The estimation method is *OLS* for all regressions:

Scheme 3.1 Stages of the analysis

Stage 1 – definition of goals, hypothesis and variables

The definition of the variables is crucial. The following variables are defined:
fundamental return (R_t^F); noise return (R_t^N); behavioral return (R_t^B).
The goals are:
1. Presenting normative and behavioral approaches to asset pricing and comparing them.
2. Describing and comparing empirical findings on nonfundamental component as well as on normative, behavioral and unified models.
3. Proposing and testing the mechanism allowing pricing capital assets, which can be used in investment decision process.
4. Comparing the proposed mechanism to existing models and checking whether it has more predictive power than existing models.

The hypotheses are:
H1: Deviation components hypothesis.
H2: Explanatory performance hypothesis.
H3: Significance hypothesis.
The data sample includes over 3800 daily observations from 14 stocks of TA35 index (Israel) and over 4000 daily observations from 50 stocks of NASDAQ 100 index (USA).

Stage 2 – the comparison of models derived from PCA

The following regressions will be tested:
- For the technical approach: $RIRF_t = R_t^N$;
- For the behavioral approach: $RIRF_t = R_t^B$;
- For the unified model: $RIRF_t = (R_t^N + R_t^B)$.

Comparison between the examined approaches is done.

Stage 3 – the comparison of the integrated models

Analysis of the regressions' results are as follows:
- For the technical approach: $RIRF_t = R_t^F + R_t^N$;
- For the behavioral approach: $RIRF_t = R_t^F + R_t^B$;
- For the unified model: $RIRF_t = R_t^F + (R_t^N + R_t^B)$.

Further, comparison between the examined approaches will be applied.

Stage 4 – the conclusions

Necessary conclusions are made.

Source: Own work

- For the technical approach: $RIRF_t^i = R_t^N$;
- For the behavioral approach: $RIRF_t^i = R_t^B$;
- For the unified model: $RIRF_t^i = (R_t^N + R_t^B)$.

The results will be compared.

In the next stage I investigate the indicators derived from the *PCA* and relevant fundamental components. The estimation method here is also *OLS* based on the following equations:

- For the technical approach: $RIRF_t^i = R_t^F + R_t^N$;
- For the behavioral approach: $RIRF_t^i = R_t^F + R_t^B$;
- For the unified model: $RIRF_t^i = R_t^F + (R_t^N + R_t^B)$.

In the last stage I conclude the study.

3.2.1. Definition of variables

Dependent Variable ($\boldsymbol{RIRF_t^i}$)

Many studies apply a stock return in excess of the *risk–free* rate as a dependent variable, assuming the *risk–free* return is certain. Despite the fact that the question of validity for such assumption is opened, this study applies the same approach defining stock excess return as:

$$RIRF\,_t^i = R_t^i - R_t^f. \tag{3.2}$$

where:

R_t^i – current price of a security i at time t;

R_t^f – risk–free return at time t;

Fundamental component ($\boldsymbol{R_t^F}$)

The fundamental price (F_t) as one conducted by fully rational investors follows a random walk with new information parameter as i_{t+1}, i.e. $F_{t+1} = F_t + i_{t+1}$. At a given point of time, the actual market price is known, while the next–day price is not. According to the normative theory, the fundamental price is also the market price and a firm intrinsic value, therefore it is the present value of discounted expected payoffs – dividends, which is given by a simple equation as follows:

$$P_t^i = \frac{P_{t+1}^{i,F} + D_{t+1}^i}{R_{t+1}^i}, \tag{3.3}$$

where:

P_t^i – current market price of a security i;

$P_{t+1}^{i,F}$ – future fundamental price of a security i;

D_{t+1}^i – dividend payment for a security i;

R_{t+1}^i – required rate of return for a security i; $R_{t+1}^i = 1 + k$.

Let us assume that the chosen period is 1 trading day. For such short period of time a dividend payment should not be expected. Instead, the payments are known when the announcement is released. Day after the announcement, future price $P_{t+1}^{i,F}$ will be updated such a way to offset the dividend payments. From here, dividend payment has no influence on daily fundamental prices, but the future price should be unadjusted to the dividend payments instead.

At the time $t = 0$, the future price is unknown to an investor and hence, the required rate of return R_{t+1}^{i} should be replaced by expected rate of return $E(R_t^i)$ (Markowitz, 1952). As long as the fundamental price is conducted by fully rational investors, consistent with the *EMH*, it is reasonable to assume that only rational–based investors determine it according to a rational–based model such as the *APT*, the Fama–French three/five–factor models or the *CAPM*. Since the dependent variable is *stock excess return*, all of the fundamental models should be devoid of *risk–free* rate. In the case of the *CAPM*, the required rate of return can be written as follows:

$$R_t^F = \beta_i\big(E(R_t^M) - R_t^f\big). \tag{3.4}$$

where:

R_t^F – fundamental return of a security i;

$E(R_t^M)$ – market return;

This allows creation of **daily** fundamental returns without any additional book data. In order to evaluate coefficients like *beta* of the *CAPM*, adoptive approach is applied. The expected returns of the *CAPM* (and other normative models) are evaluated due to past information and so the *beta* coefficient. The future is uncertain and to evaluate, for example, the beta at $t = 4$, three previous known observations are used. At the time $t = 5$, the *beta* is updated with additional certain observation at $t = 4$ and so on:

$$\beta_{t+1} = \frac{cov\big(R_{t+1}^i; R_{t+1}^m\big)}{var(R_{t+1}^m)} \qquad \beta_{t+2} = \frac{cov\big(R_{t+2}^i; R_{t+2}^m\big)}{var(R_{t+2}^m)} \qquad \beta_{t+3} = \frac{cov\big(R_{t+3}^i; R_{t+3}^m\big)}{var(R_{t+3}^m)}$$

The same method is applied to evaluate coefficients of Fama–French three/five–factor models:

$$R_t^F = \big[\beta_3(E(R_t^M) - R_t^f) + s_i \cdot SMB + h_i \cdot HML\big] + r_i \cdot RMW + c_i \cdot CMA + \alpha. \tag{3.5}$$

Theoretically, any fundamental model can be applied to determine "rational" returns. Here, the *CAPM* is applied as the basic normative one and the Fama–French five–factor model is applied as the newest and the most acceptable among the normativists.

Noise component (R_t^N)

The noise price (N_t) is conducted by rational investors, who are similar to the fully rational investors in the sense of the *EMH*. They have the same decision–making basis, though their decision factors are technical analysis indicators instead the normative tools. Already Lease, Lewellen and Schlarbaum (1974) emphasize that economic advisory leads to suboptimal investment decisions. However, the traders still believe that economic advisory or technical analysis may give them an advantage to make a better investment decision and act as this information is true. Such activity creates noise from one side and potential to extra profits for fully rational investors from the other side.

Measuring scientifically the real influence of technical indicators on the stock excess returns is not easy since buy/sell signals do not build a time series, though several approaches are applied to test effectiveness of such indicators. One of the most applicable is the *Principal Component Analysis* (*PCA*). Neely et al (2014) compose a technical trading index based on 14 rules created from 3 simple and popular indicators, like moving averages, momentum or on–balance volume, and compare it to similar index created from 14 macroeconomic variables. Sadaqat and Butt (2016) incorporate several technical indicators into sentiment index through the same *PCA*. The *PCA* allows reducing large numbers of explanators with losing as less as possible of its information. It has additional advantages like eliminating multicollinearity that time series often suffer from. It is useful especially in cases when the meaning of every single variable is not important, but the influence of all variables shrank together. According to Neely et al (2014), all 14 trading rules can be divided into 3 groups regarding to its origin:

- **volume;**
- **momentum;**
- **trend.**

I adopt the methodology of Neely et al (2014) with modified variables and create 15 trading rules on the daily basis as follows:

(1) <u>*Simple Moving Averages* (*SMA*)</u>:

Moving averages in various forms are extremely popular in the literature. To generate a buy/sell signal, short–run and long–run moving averages are in use at the same time. Two short–run (s) and three long–run (l) moving averages are applied to examine their influence on the $RMRF_t^i$. During the construction of *Technical Analysis* (TA_t) indicators, 6 pairs regarding short–run *SMA,* $s = 1, 5$ days and long–run *SMA,* $l = 10, 20, 50$ days are created:

- SMA(1/10) => SMA_t^{110};
- SMA(1/20) => SMA_t^{120};
- SMA(1/50) => SMA_t^{150};
- SMA(5/10) => SMA_t^{510};
- SMA(5/20) => SMA_t^{520};
- SMA(5/50) => SMA_t^{550}.

The period lengths differ from those used in the original study of Neely et al (2014) since here I implement real life practitioner's approach. The underlying assumption is that prices can be influenced only if all practitioners use the same tools and hence, see the same picture. However, overall technique is similar to the original one of Neely et al (2014).

The original sell/buying signals methodology can be traced back to Neftci and Policano (1984). The signals for security i are created by comparing two moving averages, where a signal may imply $S_{i,t} = \{1; 0\}$:

$$S_{i,t} = \begin{cases} 1 \; if \; MA_{s,t} \geq MA_{l,t} \\ 0 \; if \; MA_{s,t} < MA_{l,t} \end{cases}.$$

(2) <u>*Momentum–Rate Of Change (ROC):*</u>

Momentum generates a signal regarding to some past price, $t - m$ periods ago. Three momentum indicators are applied in the construction of *Technical Analysis* indicators and those are different from the original indicators of Neely et al (2014). Here it is applied the *ROC* for security i which is created by comparing two security's prices from current and past period $t - m$, when $m = 10, 20, 50$:

$$ROC_t = \frac{P_t - P_{t-m}}{P_{t-m}} * 100\%;$$

Further, simple moving averages are applied on the series of *ROC*, where a signal either $S_{i,t} = \{1; 0\}$, while $S_{i,t}$ Is defined by the next system:

$$S_{i,t} = \begin{cases} 1 \; if \; MA_{s,t}^{ROC} \geq MA_{l,t}^{ROC} \\ 0 \; if \; MA_{s,t}^{ROC} < MA_{l,t}^{ROC} \end{cases}.$$

Thus, 3 variables for *ROC* regarding to length of $s = 5$ days and of $l = 10, 20, 50$ days are created:

- Moving Average on the Rate Of Change MAROC(5/10) => $MAROC_t^{510}$;
- Moving Average on the Rate Of Change MAROC(5/20) => $MAROC_t^{520}$;
- Moving Average on the Rate Of Change MAROC(5/50) => $MAROC_t^{550}$.

(3) *On–Balance Volume (OBV)*:

Additional indicator, from the volume group, is *OBV*, which is developed by Joe Granville (1963). Such indicator is also popular in the literature, see for example Sullivan et al (1999) or Ng et al (2014). The indicator measures positive and negative volume flow and is calculated as follows:

$$If \; P_t > P_{t-1}, then \; OBV_t = \; OBV_{t-1} + Volume_t,$$

$$If \; P_t < P_{t-1}, then \; OBV_t = \; OBV_{t-1} - Volume_t,$$

$$If \; P_t = P_{t-1}, then \; OBV_t = \; OBV_{t-1}.$$

The signals are generated as applied in Sullivan et al (1999) and may imply as follows: $S_{i,t} = \{1; 0\}$, when $S_{i,t}$ is defind by next system:

$$S_{i,t} = \begin{cases} 1 \; if \; MA_{s,t}^{OBV} \geq MA_{l,t}^{OBV} \\ 0 \; if \; MA_{s,t}^{OBV} < MA_{l,t}^{OBV} \end{cases}.$$

When the *OBV* series is created, moving averages are applied on the on–balance volumes in short–run and long–run terms. Thus, 6 pairs of volumes regarding to length of $s = 1, 5$ days and of $l = 10, 20, 50$ days are created:

- Moving Average on ON BALANCE VOLUME(MA1/10OBV) => $MA110_t^{OBV}$;
- Moving Average on ON BALANCE VOLUME(MA1/20OBV) => $MA120_t^{OBV}$;
- Moving Average on ON BALANCE VOLUME(MA1/50OBV) => $MA150_t^{OBV}$;
- Moving Average on ON BALANCE VOLUME(MA5/10OBV) => $MA510_t^{OBV}$;
- Moving Average on ON BALANCE VOLUME(MA5/20OBV) => $MA520_t^{OBV}$;
- Moving Average on ON BALANCE VOLUME(MA5/50OBV) => $MA520_t^{OBV}$.

Neely et al (2014) argue that also lagged values of the same variables should be included as *Technical Analysis* indicators since some signals may influence next–day returns. However, if the subject is daily price formation, then applying the lagged values is not necessary. Thus, 15 variables are enough to construct the necessary indicators.

Behavioral component (R_t^B)

According to Szyszka (2009) the behavioral price (B_t) is a generalization of 3 common psychological biases. Theoretically, it is possible to generalize **all** psychological biases exactly in the same manner. On the aggregative level such generalization should lead to the *market sentiment* and hence should be measurable. According to Brown and Cliff (2004) or Baker and Wurgler (2006) the market sentiment can be measured directly, with survey, or indirectly, with observable market data. They suggest to construct a *sentiment index* through the *PCA* though they apply the index to whole market, but not to single securities. Contrary, Sadaqat and Butt (2016) suggest to construct a *sentiment index*, using several indicators from technical analysis, where such index turns to suitable with single securities.

In contrast to the method acceptable in the literature, in this study the principle components retained directly to further estimation based on Kaiser's (1960) rule. Hence, several components can be retained in order to prevent loss of information as less as possible. According to Trzcinka (1986) only the first and the one eigenvalue dominates the covariance matrix, hence it has the greatest importance. Thus, in the literature only first principle component is in use to construct composite sentiment index. Indeed, the first principal component captures the largest part of original variation, though it can be not enough; several other components can be loaded with additional useful information that is not included in the first component. From here, Kaiser's rule allows to retain more components and by this to capture even larger part of original variation. Similar to the case of technical analysis, due to daily price formation and contrary to the literature, applying the lagged values is not necessary, hence the two–step procedure is omitted.

There is no concrete instruction or agreement in the literature how many variables should be included in the sentiment measure. Baker and Wurgler (2006) apply 6 variables and this is the most common number. However, Hudson and Green (2015) apply 8 variables while Yang and Gao (2014) apply only 4. With it Baker and Wurgler (2006) argue that finite number of the variables is less important, the only subject a researcher should worry about is that the number of the variables will be sufficiently large, i.e. more than 3. In the thesis I apply 5 variables.

According to Brown and Cliff (2004), direct and indirect measurements of the sentiment reveals similar results, therefore both are good for the empirical study. Here indirect sentiment measure is applied by constructing *Sentiment Indicators* (SI_t) in a favor of Sadaqat and Butt (2016). Four original indicators, used by Sadaqat and Butt (2016), are applied to measure the *Sentiment Indicators*. Those are:

(1) *Relative Strength Index* (*RSI*):

The index is developed by J. Welles Wilder (1978) to measure the speed and change of price movements. The *RSI*, which is momentum oscillator, is very popular among scientists and among traders. The justification for use of the index in constructing the *Sentiment Indicators* can be found in Yang and Zhou (2015) for the Chinese market or in Hudson and Green (2015) for the UK market. According to Wilder (1978), when the value of *RSI* is above 70 the market is overbought and oversold otherwise. It is calculated in 2 steps as follows:

$$RS_t = \frac{Average\ Gain}{Average\ Loss} \rightarrow RS_t = \frac{\sum_{t=1}^{14}\left(P_{t-j} - P_{t-j-1}\right)^+}{\sum_{t=1}^{14}\left|P_{t-j} - P_{t-j-1}\right|} \tag{1}$$

$$RSI_t = 100 - \frac{100}{1 + RS_t} \tag{2}$$

(2) *Money Flow Index* (*MFI*):

The index is introduced by Quong and Soudak (1989) in attempt to create a measurement for buying and selling pressures and sometime called Weighted Relative Strength Index. The justification for using this index in constructing the sentiment index can be found in Chen, Chong, and Duan (2010) for the Chinese market or in Hudson and Green (2015) for the UK market. The index is calculated in 4 steps for the period of 14 days:

$$TP_t = \frac{H_t + L_t + P_t}{3}, \tag{1}$$

where:

TP_t – typical price at time *t*;

H_t – high daily price at time *t*;

L_t – low daily price at time *t*;

P_t – closing price at time t.

$$Money\ Flow_t = TP_t * Volume_t, \tag{2}$$

$$Money\ Ratio\ (MR_t) = \frac{Positive\ Money\ Flow_t}{Negative\ Money\ Flow_t}, \tag{3}$$

$$MFI_t = 100 - \frac{100}{1 + MR_t}. \tag{4}$$

(3) <u>*Psychological Line Index* (*PLI*)</u>:

The index was introduced by Kim And Ha (2010) and designed to measure short–term reversals in the markets. It attempts to answer how to quantify the obvious mood of the market and to detect undertones for a trend change. The index is calculated as a ratio of number of periods where previous price is lower than the current price to total period. When the *PLI* is about 75 the market is overbought, otherwise it is oversold. The justification for using this indicator can be found in Yang and Gao (2014) and Yang and Zhou (2015). The *PLI* is given by the following equition:

$$PLI_t = 100 * \frac{t}{T},$$

where:

t – number of periods where previous price is lower than the current price;

T – total period, where the standard is 25 days.

(4) <u>*Adjusted Share Turnover Ratio* (*ATR*)</u>:

The turnover ratio is very popular in the literature. Scheinkman and Xiong (2003), Brown and Cliff (2004) and Baker and Wurgler (2006) integrate it in their composite sentiment index. The ratio measures a security's liquidity. However, Yang and Zhang (2014) or Yang and Zhou (2015) suggest to apply adjusted ratio since the *STR* does not demonstrate whether the investor sentiment is optimistic or pessimistic. For this reason, the ration is multiplied on return ratio to determine its sign:

$$ATR_t = \left(\frac{R_t}{|R_t|}\right)\left(\frac{Trading\ Volume_t}{Shares\ Outstanding_t}\right).$$

where:

R_t – return at time t; in a case of $R_t = 0$, the return ratio is signed as positive.

(5) *Dollar Volume Approximation (DVA)*:

DVA is additional sentiment indicator that does not appear in the study of Sadaqat and Butt (2016). However, volume, or more generally liquidity, is the important sentiment measure. Some studies, like Liao, Huang, and Wu (2011), even suggest to apply the pure trading volume (*VOL*) during construction of sentiment index. In contrast, I apply the volume in the means of money. For this purpose, the trading volume is multiplied by the closing price (P_t) which can be a good approximation for the dollar volume measurement:

$$DVA_t = VOL_t * P_t.$$

3.2.2. Goals and hypotheses

Motives

Two main financial theories — normative and behavioral — exist side by side, describing the same financial phenomena of capital asset pricing by different explanations that even contradict each other. None of them has enough evidence to refute the competitive one, while each of them has sufficient evidence to support its own view. Both theories are good, but seems like not good enough. Otherwise, only one theory would give an appropriate description of the financial reality. However, instead of disputing which theory is better, it is possible to integrate one theory into another and make a step to creation of a platform for one **unified**, **integrated** and **solid** financial theory. I believe that integration of the best achievements of both theories will lead to better results and to more accurate financial reality description. From here, there are two motives in creation of the Unified Capital Asset Pricing Model:

(1) **Unification:** integration of the rational–based and non–rational–based approaches into one pricing mechanism, which is the Unified Capital Asset Pricing Model. Traditional and behavioral approaches are integrated into one model.

(2) **Universality:** availability of using the asset pricing mechanism by both rational–based and non–rational–based individuals the same manner — rationality independence, which means it is necessary to consider all possible types of investors. The mechanism allows seeing the same "big picture" (Fama, 1998) by all participants of a market.

Goals

For decades, the debates of the nature of the aberrant from the fundamental price (F) is either rational **or** behavioral. I think that the problem is similar to a coin, when each concept may see only one side of it. Switching the coin 90^o will allow the concepts see each other. For this reason, the integration and the unification can be a solution. The main purpose of this PhD thesis is to investigate whether the integration and the unification indeed can be done and it even has a better explanatory power than existing approaches suggest. According to the assumption that non–fundamental price (NF) contains both noise (N) **and** behavioral (B) components, the goal is to investigate whether non–fundamental price indeed can be explained in the noise and behavioral terms. Reaching such conclusion is possible only by obtaining statistically significant results for all components of the Unified Capital Asset Pricing Model. Moreover, as I believe the Unified Capital Asset Pricing Model potentially has a better explanatory power, the goal is to investigate whether the model surpasses existing traditional and behavioral explanations. This can be reached by comparing the effects of the Unified Capital Asset Pricing Model with those of traditional and behavioral approaches.

From here the main goal of this thesis is:

Building the model of capital asset pricing, which has a predictive power and is more consistent with real economic data than existing normative and behavioral models.

There are 4 sub–goals, which are:

1. Presenting normative and behavioral approaches to asset pricing and comparing them.
2. Describing and comparing empirical findings on non–fundamental component as well as on normative, behavioral and unified models.
3. Proposing and testing the mechanism allowing pricing capital assets, which can be used in investment decision process.
4. Comparing the proposed mechanism to existing models and checking whether it has more predictive power than existing models.

Hypotheses

Behavioral models as well as the normative models both fail to fully reflect economic reality. Mostly those models deal good with a given asset pricing anomaly, but they lose their ability in explaining the overall returns. If both theories fail to do so, theoretically it is possible to propose a model, which has a potential to fill the gap of existing models. As the unified model assumes the non–fundamental price is an integration of normative and behavioral approaches, the main hypothesis of this PhD thesis is: **there exists a possibility to create an asset pricing mechanism which combines normative and behavioral approaches to asset pricing and has a better explanatory power than existing models**. In other words, there are some variables which make the proposed asset pricing model more accurate than the existing models.

The hypotheses are derived from the goals. From the definitions of the variables, the noise is measured by *Technical Analysis* (*TA*) indicators and the behavioral component is measured by *Sentiment Indicators* (*SI*). From here the hypotheses are:

H1: Deviation components hypothesis

The deviation from fundamental price can be explained in terms of noise and behavioral components, i.e. in terms of *Technical Analysis* index and *Sentiment Indicators*.

H2: Explanatory performance hypothesis

The Unified Capital Asset Pricing Model has a better explanatory power of the deviation from the fundamental price than traditional or behavioral approaches separately, which is expressed in higher R^2. It is obtained when $R^2 \geq 0.5$ for the fully integrated regressions.

H3: Significance hypothesis

All the components of the Unified Capital Asset Pricing Model are statistically significant.

3.2.3. Analysis procedure

The model assumes that the nature of the aberrant from the fundamental price is both rational and behavioral and a stock excess return is composed of three returns, i.e. fundamental, noise and behavioral returns. To show the difference, it is necessary to compare the unified model with each approach separately. For this purpose, 3 regressions will be created:

(1) For the behavioral approach: $RIRF_t^i = R_t^F + R_t^B$;

(2) For the technical approach: $RIRF_t^i = R_t^F + R_t^N$;

(3) For the unified approach: $RIRF_t^i = R_t^F + (R_t^N + R_t^B)$.

For the behavioral approach (regression (1)), first RIRF is regressed on sentiment indicators to determine if it may ever have any predictive ability. Next, through the *PCA*, based on a

Kaiser's (1960) rule of eigenvalues greater or equal to 1, appropriate number of factors *SI* will be retained. Before applying the *PCA*, all the behavioral variables will be standardized. According to Yang and Zhou (2015) one might object that the *PCA* cannot distinguish between a common sentiment component and a common *CAPM* component. To remove the common dependence of the parsimonious investor sentiment measurements on the *CAPM* factor, in the case of Israeli market, devoid of risk–free rate *CAPM* model will be regressed on retained *SIs* in order to isolate the influence of sentiment indicators on the *CAPM* and to pull the residuals (RES_t^{SI}) from the regression. The main reason to pull the residuals is the fact that investigated stocks may appear in calculation of the *CAPM*. Last, $RIRF_t^i$ will be regressed on *SIs* and the RES_t^{SI}, but before, $RIRF_t^i$ will be regressed on *SIs* to see its pure contribution:

$$CAPM_t = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + u_t \rightarrow RES_t^{SI};$$

$$RIRF_t^i = \alpha + \beta_1 RES_t^{SI} + \beta_k \sum_{k=1}^{K} SI_{k,t} + \varepsilon_t, \tag{3.6}$$

$$K = \{k_1, \dots, k_n | eigenvalue \geq 1\}.$$

In the case of the US market, considering the argument of Yang and Zhou (2015) the same procedure will be done for every single factor of Fama–French five–factor model:

$$E(R_t^M) - R_t^f = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + u_t \rightarrow RES_t^{MKT};$$

$$SMB_t = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + u_t \rightarrow RES_t^{SMB};$$

$$HML_t = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + u_t \rightarrow RES_t^{HML};$$

$$RMW_t = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + u_t \rightarrow RES_t^{RMW};$$

$$CMA_t = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + u_t \rightarrow RES_t^{CMA};$$

$$RIRF_t^i = \alpha + \beta_1 RES_t^{MKT} + \beta_2 RES_t^{SMB} + \beta_3 RES_t^{HML} + \beta_4 RES_t^{RMW} + \beta_5 RES_t^{CMA} + \tag{3.7}$$

$$+\beta_k \sum_{k=1}^{K} SI_{k,t} + \varepsilon_t,$$
$$K = \{k_1, \dots, k_n | eigenvalue \geq 1\}.$$

For the technical approach (regression (2)), technical analysis indicators will be used. Neely et al (2014) give a legitimation to use the same approach as used for retaining sentiment components. Much like in previous procedure, technical analysis components *TA* will be retained. Next, in the case of Israeli market, devoid of risk–free rate *CAPM* model will be regressed on the retained *TAs* in order to isolate the influence of technical indicators on the *CAPM* and to pull the residuals (RES_t^{TA}) from the regression. Last, $RIRF_t^i$ will be regressed on *TAs* and the RES_t^{TA}, but before, $RIRF_t^i$ will be regressed on *TAs* to see its pure contribution:

$$CAPM_t = \alpha + \beta_k \sum_{k=1}^{K} TA_{k,t} + u_t \rightarrow RES_t^{TA};$$
$$RIRF_t^i = \alpha + \beta_1 RES_t^{TA} + \beta_k \sum_{k=1}^{K} TA_{k,t} + \varepsilon_t, \tag{3.8}$$
$$K = \{k_1, \dots, k_n | eigenvalue \geq 1\}.$$

In the case of the US market, the same procedure will be done for every single factor of Fama–French five–factor model:

$$E(R_t^M) - R_t^f = \alpha + \beta_k \sum_{k=1}^{K} TA_{k,t} + u_t \rightarrow RES_t^{MKT};$$
$$SMB_t = \alpha + \beta_k \sum_{k=1}^{K} TA_{k,t} + u_t \rightarrow RES_t^{SMB};$$
$$HML_t = \alpha + \beta_k \sum_{k=1}^{K} TA_{k,t} + u_t \rightarrow RES_t^{HML};$$
$$RMW_t = \alpha + \beta_k \sum_{k=1}^{K} TA_{k,t} + u_t \rightarrow RES_t^{RMW};$$
$$CMA_t = \alpha + \beta_k \sum_{k=1}^{K} TA_{k,t} + u_t \rightarrow RES_t^{CMA};$$
$$RIRF_t^i = \alpha + \beta_1 RES_t^{MKT} + \beta_2 RES_t^{SMB} + \beta_3 RES_t^{HML} + \beta_4 RES_t^{RMW} + \beta_5 RES_t^{CMA} + \tag{3.9}$$

$$+\beta_k \sum_{k=1}^{K} TA_{k,t} + \varepsilon_t,$$

$$K = \{k_1, \dots, k_n | eigenvalue \geq 1\}.$$

There are no lagged values for the technical indicators since the price formation is daily and depends only on a buy/sell signal in the particular time t.

Finally, the integrated (regression (3)) will be applied for the unified model by exactly the same technique and the results for all regressions will be compared with further conclusions. For the Israeli market:

$$CAPM_t = \alpha + \beta_k \sum_{k=1}^{K} ALL_{k,t} + u_t \rightarrow RES_t^{ALL};$$

$$RIRF_t^i = \alpha + \beta_1 RES_t^{ALL} + \beta_k \sum_{k=1}^{K} ALL_{k,t} + \varepsilon_t, \tag{3.10}$$

$$K = \{k_1, \dots, k_n | eigenvalue \geq 1\}.$$

For the US market:

$$E(R_t^M) - R_t^f = \alpha + \beta_k \sum_{k=1}^{K} ALL_{k,t} + u_t \rightarrow RES_t^{MKT};$$

$$SMB_t = \alpha + \beta_k \sum_{k=1}^{K} ALL_{k,t} + u_t \rightarrow RES_t^{SMB};$$

$$HML_t = \alpha + \beta_k \sum_{k=1}^{K} ALL_{k,t} + u_t \rightarrow RES_t^{HML};$$

$$RMW_t = \alpha + \beta_k \sum_{k=1}^{K} ALL_{k,t} + u_t \rightarrow RES_t^{RMW};$$

$$CMA_t = \alpha + \beta_k \sum_{k=1}^{K} ALL_{k,t} + u_t \rightarrow RES_t^{CMA};$$

$$RIRF_t^i = \alpha + \beta_1 RES_t^{MKT} + \beta_2 RES_t^{SMB} + \beta_3 RES_t^{HML} + \beta_4 RES_t^{RMW} + \beta_5 RES_t^{CMA} +$$

$$+\beta_k \sum_{k=1}^{K} ALL_{k,t} + \varepsilon_t, \tag{3.11}$$

$$K = \{k_1, \dots, k_n | eigenvalue \geq 1\}.$$

3.2.4. Data

There are 2 samples used in the study. One stands for the Israeli market and one for the US market, when both for same period length. Those 2 markets can be seen as one opposite to another: very big US market vs very small Israeli one; high degree of government control and restrictions on the Israeli market vs freer market in the US. The sample period begins from 1/2/2001 and ends in 1/2/2017 (16 trading years). Despite equal period length, the total number of observations varies. There are 3925 observations recorded for the Israeli market and for the US market it is recorded 4025 observations since there are less trading days in Israeli market per year. In order to construct the variables, one additional year was used: 3/1/2000 – 31/1/2001.

The whole period is divided into 3 subperiods:

1) 1/2/2001 – 1/2/2006 => 5 trading years.
2) 2/2/2006 – 1/2/2011 => 5 trading years.
3) 2/2/2011 – 1/2/2017 => 6 trading years

The second subperiod includes the subprime mortgage crisis. The third subperiod is longer than two previous periods to validate whether the period length may influence the results.

All the relevant data for the Israeli market was pulled from the official Tel Aviv Stock Exchange site[28]. As a result of lack of full data sets, the sample contains only 14 companies from the TL35 index (today), which includes 35 most capitalized companies on the market and replaces TL25 index from late February, 2017. Hence, the market return is described by the TL25 and the risk–free parameter is described by Short Term Treasury Bill Index (MAKAM).

To construct the US market sample several data sources were used. Again, since a lack of full data sets, the sample includes 50 companies from NASDAQ100 index and most of relevant data was pulled from the Yahoo! Finance site[29]. The data for shares outstanding was pulled from Bloomberg.com site and the data for the five/three factor models as well as for risk–free parameter was pulled from the official Kenneth French site[30]. The full list of the companies used for both markets can be seen in Table 3.1 in the Annex.

[28] https://info.tase.co.il/Eng/MarketData/Stocks/MarketData/Pages/MarketData.aspx
[29] https://finance.yahoo.com/?guccounter=1
[30] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

3.3. Results of tests of the models

3.3.1. Results for the single models

First, we see that pure technical and sentiment variables (before the *PCA*) have any predictive ability. Table 3.2a and Table 3.3a summarize the results for the distribution of the coefficients from regressions based on pure or direct technical and sentimental variables for the US and Israeli markets respectively by the level of significance. Tables 3.2b and Table 3.3b in the Annex demonstrate average values of the coefficients based on the equations (3.12) and (3.13) respectively.

The regression equation for technical analysis variables are as the follows:

$$\begin{aligned} RIRF_t^i = \alpha &+ \beta_1 SMA_t^{110} + \beta_2 SMA_t^{120} + \beta_3 SMA_t^{150} + \beta_4 SMA_t^{510} + \beta_5 SMA_t^{520} + \\ &+ \beta_6 SMA_t^{550} + \beta_7 MAROC_t^{510} + \beta_8 MAROC_t^{520} + \beta_9 MAROC_t^{550} + \\ &+ \beta_{10} MA110_t^{OBV} + \beta_{11} MA120_t^{OBV} + \beta_{12} MA150_t^{OBV} + \\ &+ \beta_{13} MA510_t^{OBV} + \beta_{14} MA520_t^{OBV} + \beta_{15} MA550_t^{OBV} + \varepsilon_t. \end{aligned} \tag{3.12}$$

Table 3.2a, Panel A to D, demonstrates distribution of number of original/direct technical analysis variables by level of significance based on the equation (3.12) for the US and Israeli markets. It is possible to see that the vast majority of the coefficients[31] is significant at the level of 1% in both markets. For the whole examined period there are 84% significant coefficients at highest level on the US market and 74% on the Israeli market. The largest number of significant coefficients at all levels appears in Panel A for both markets. It is about 95% for the US market and about 89% for the Israeli market. Similarly, the lowest number of significant coefficients at all levels appears in Panel C for both markets that reflects affection of the sub–prime mortgage crisis. It is about 77% on the US market and about 67% on the Israeli market. Panel B includes similar results as in Panel C. Panel D demonstrates some improvement in significance relatively to Panels C and B, while Panel D includes larger sample, 6 trading years against 5 in Panels C and B. It is about 84% of all significant coefficients on the US market and about 75% on the Israeli market in Panel D. This suggests that **with time the significance of the coefficients as well as its quality increases**. The lowest significance is recorded for moving averages with long periods like SMA_t^{520}, SMA_t^{550} or $MA520_t^{OBV}$ and $MA550_t^{OBV}$. From other side $MAROC_t^{510}$, $MAROC_t^{520}$ and $MAROC_t^{550}$ are always found at the highest level of significance, suggesting that **momentum plays a greater role than other technical variables**.

[31] Here and further does not refer to the intercepts.

Table 3.2a Distribution of number of original/direct technical analysis variables by level of significance

$$RIRF_t^i = \alpha + \beta_1 SMA_t^{110} + \beta_2 SMA_t^{120} + \beta_3 SMA_t^{150} + \beta_4 SMA_t^{510} + \beta_5 SMA_t^{520} + \beta_6 SMA_t^{550} + \beta_7 MAROC_t^{510} + \beta_8 MAROC_t^{520} + \beta_9 MAROC_t^{550} + \beta_{10} MA110_t^{OBV} + \beta_{11} MA120_t^{OBV} + \beta_{12} MA150_t^{OBV} + \beta_{13} MA510_t^{OBV} + \beta_{14} MA520_t^{OBV} + \beta_{15} MA550_t^{OBV} + \varepsilon_t$$

	Signif.	Const.	SMA_t^{110}	SMA_t^{120}	SMA_t^{150}	SMA_t^{510}	SMA_t^{520}	SMA_t^{550}	$MAROC_t^{510}$	$MAROC_t^{520}$	$MAROC_t^{550}$	$MA110_t^{OBV}$	$MA120_t^{OBV}$	$MA150_t^{OBV}$	$MA510_t^{OBV}$	$MA520_t^{OBV}$	$MA550_t^{OBV}$	TOTAL
Panel A: 1/2/2001-1/2/2017	1%	50	50	50	50	50	34	42	50	50	50	48	26	39	50	12	29	680
	5%						10	4				2	17	7		13	8	61
US market - 50 securities	10%						2	2					3	2		8	6	23
	10%>						4	2					4	2		17	7	36
	1%	14	14	14	14	14	8	5	14	14	14	12	8	6	13	3	2	169
Israeli market - 14 securities	5%						4	3				2	2	1	1	2	1	16
	10%							3					2	2		2	4	13
	10%>						2	3					2	5		7	7	26
Panel B: 1/2/2001-1/2/2006	1%	50	38	32	36	47	11	16	50	50	50	27	5	10	36	1	10	467
	5%		9	10	9	2	10	14				8	8	15	6		7	98
US market - 50 securities	10%		1	4	2		4	6				6	7	8	2	9	8	57
	10%>		2	4	3	1	25	14				9	30	17	6	40	25	169
	1%	13	12	10	6	12	3	1	13	14	13	2	4		7	2	1	113
Israeli market - 14 securities	5%	1	1	2	1	2	1	2	1		1	3	2	3	2	2	1	25
	10%		1	1	3		3	3				4	1	3	1	1	1	22
	10%>			1	4		7	8				5	7	8	4	9	11	64
Panel C: 2/2/2006-1/2/2011	1%	50	40	25	39	46	4	15	50	48	50	25	6	15	29	4	6	452
	5%		6	19	7	4	15	9				8	8	10	12	10	11	119
US market - 50 securities	10%		2	4	1		11	7				6	7	8	4	4	5	59
	10%>		2	2	3		20	19		2		11	29	17	5	32	28	69
	1%	14	11	7	8	11	2		14	14	14	5		1	10		1	112
Israeli market - 14 securities	5%		1	3	4	2	4	3				2	5	1	1		1	27
	10%			2	1		3	2				1	4	1	2	1		17
	10%>		2	2	1	1	5	9				6	5	11	1	13	12	68
Panel D: 2/2/2011-1/2/2017	1%	50	37	42	45	48	16	29	50	50	50	25	14	17	38	6	10	527
	5%		12	6	3	1	10	8				12	14	12	5	11	10	104
US market - 50 securities	10%				2	1	7	3				6	9	7	3	4	6	48
	10%>		1	2			17	10				7	13	14	4	29	24	36
	1%	14	14	9	10	13	6	4	14	14	14	6	1	2	8	2	1	132
Israeli market - 14 securities	5%			2		1	1	3				5	4	1	6		1	24
	10%			2	3		2	1				1	1	3		1	1	15
	10%>			1	1		5	6				2	8	8		11	11	53

Source: Own analysis

The regression for sentiment variables is based on the following equation:

$$RIRF_t^i = \alpha + \beta_1 RSI_t + \beta_2 MFI_t + \beta_3 PLI_t + \beta_4 ATR_t + \beta_5 DVA_t + \varepsilon_t. \quad (3.13)$$

Table 3.3a, Panel A to D, demonstrates distribution of number of original/direct sentiment variables by level of significance based on the equation (3.13).

Table 3.3a Distribution of number of original/direct sentiment analysis variables by level of significance

$RIRF_t^i = \alpha + \beta_1 RSI_t + \beta_2 MFI_t + \beta_3 PLI_t + \beta_4 ATR_t + \beta_5 DVA_t + \varepsilon_t$

	Signif.	Const.	RSI_t	MFI_t	PLI_t	ATR_t	DVA_t	TOTAL
Panel A: 1/2/2001-1/2/2017	1%	43	50	46	46	47	25	257
	5%	4					7	11
US market - 50 securities	10%			1			4	5
	10%>	3		3	4	3	14	27
	1%	14	14	14	13	14	8	77
Israeli market - 14 securities	5%						1	1
	10%						1	1
	10%>				1		4	5
Panel B: 1/2/2001-1/2/2006	1%	33	48	46	43	50	22	242
	5%	7	2		4		7	20
US market - 50 securities	10%	1			1		2	4
	10%>	9		4	2		19	34
	1%	8	8	7	7	8	6	44
Israeli market - 14 securities	5%			1				1
	10%							
	10%>	6	6	6	7	6	8	39
Panel C: 2/2/2006-1/2/2011	1%	18	49	46	48	50	23	234
	5%	10		1	2		6	19
US market - 50 securities	10%	3	1	1			7	12
	10%>	19		2			14	35
	1%	14	14	14	10	14	9	75
Israeli market - 14 securities	5%				2		1	3
	10%							
	10%>				2		4	6
Panel D: 2/2/2011-1/2/2017	1%	30	48	47	49	50	29	253
	5%	5		1	1		6	13
US market - 50 securities	10%	3	1	1			1	6
	10%>	12	1	1			14	28
	1%	14	14	14	14	14	6	76
Israeli market - 14 securities	5%						2	2
	10%						1	1
	10%>						5	5

Source: Own analysis

Results collected in Table 3.3a demonstrate that coefficient pattern is stable for both markets in the case of the sentiment variables, as well as in the case of technical variables. It includes positive *RSI* and *ATR* against negative *MFI* and *PLI* while almost all of these variables are significant at high levels. *DVA* can be positive or negative as well as it can be significant or insignificant. *DVA* is the only indicator that is insignificant very often. That raises a question whether volume is a good sentiment indicator. The *DVA* seems not to be a good measure. However, it is necessary to incorporate all relevant variables despite of its insignificance since those variables are loaded with unique information that shouldn't be lost during the *PCA*. Other variables are insignificant extremely rare even during the period of sub–prime crisis for both markets. Similar to the case of the technical analysis, Panel A demonstrates the best performance for significance with only 5.6% insignificant coefficients for the US market and about 7% for the Israeli market. Panel B records worst significance with 10.4% insignificant coefficients for the US market and about 18.6% on the Israeli market. Panel C includes 6.5% of insignificant coefficients in the US and 8.6% in Israel. **The results indicate that during sub–prime crisis markets were more sentimental than technical**. According to Panel D, only 7% of coefficients are insignificant in Israel and 6.4% in the US. As well as in the case of the technical indicators, Panel D exhibits some improvement in significance relatively to Panel C once again suggesting that longer the period, the higher the significance and the quality of the variables.

Table 3.4 demonstrates the results of minimum, maximum and average R^2 values for the US and Israeli markets, comparing it between the technical and sentiment regressions based on the equations (3.12) and (3.13).

Table 3.4 Comparison of $\mathbf{R}^2$ values for direct variables between the US and Israeli markets

	R^2	US			Israel		
		Min	Max	Average	Min	Max	Average
Panel A: 1/2/2001-1/2/2017	Technical	0.23331	0.32878	0.29198	0.15120	0.34518	0.28903
	Sentiment	0.39764	0.69128	0.58414	0.38028	0.58609	0.50002
Panel B: 1/2/2001-1/2/2006	Technical	0.24431	0.36880	0.31191	0.16903	0.39459	0.32167
	Sentiment	0.21919	0.71974	0.59246	0.23570	0.63805	0.48253
Panel C: 2/2/2006-1/2/2011	Technical	0.25911	0.33916	0.30490	0.12701	0.36537	0.29552
	Sentiment	0.45892	0.69209	0.62357	0.37640	0.61607	0.53072
Panel D: 2/2/2011-1/2/2017	Technical	0.22359	0.36706	0.32600	0.29405	0.35250	0.32631
	Sentiment	0.54722	0.73817	0.65523	0.36947	0.63928	0.55183

Source: Own analysis

The coefficient of explanation R^2 of technical analysis is somewhat small and varies between 0.224 to 0.369 on the US market, mostly it falls between 0.270 to 0.330. However, in Panels B, C, D there are more cases with R^2 exceeding 30%. On the Israeli market in most cases R^2 exceeds 30% but from other side there are some companies with R^2 under 20%. On both markets Panel A includes a little lower R^2. The average value is close to 30% for all periods regarding both the US and Israeli markets.

On both markets regressions with sentiment variables exhibit high R^2 that easily exceeds 60% on the US market and 50% on the Israeli one. The average value stands on about 60% in the US while in Israel it is about 51%. On the first glance such predictive ability of pure sentiment variables looks very impressive, however this is a result of multicollinearity. The reason for multicollinearity comes out from the nature of the indicators themselves, since the creation of the indicators involves the same parameters. For example, *MFI* is often called volume–weighted *RSI*, hence both can be touched by the multicollinearity and so on. This problem is resolved during the *PCA*.

Generally, the pure technical as well pure sentiment variables are good to explain returns, however their ability to do so is very limited. For this reason, their integration with fundamental indicators has a potential to increase their explanatory level.

The fundamental indicators are presented by the *CAPM* for the Israeli market and by the five–factor model for the US market. Table 3.5 and Table 3.6, Panels A to D, compare the results of three–factor and five–factor models based on the following equations respectively:

$$RIRF_t^i = \alpha + \beta_1(Mkt - R^f)_t + \beta_2 SMB_t + \beta_3 HML_t + \varepsilon_t, \tag{3.14}$$

and

$$RIRF_t^i = \alpha + \beta_1(Mkt - R^f)_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 RMW_t + \beta_5 CMA_t + \varepsilon_t, \tag{3.15}$$

Table 3.5 $\mathbf{R^2}$ values of fundamental models

		R^2_{3f}	R^2_{5f}	$R^2_{5f} - R^2_{3f}$	R^2_{CAPM}
Panel A: 1/2/2001-1/2/2017	Min	0.20210	0.20757	0.00061	0.09020
	Max	0.60087	0.60917	0.07039	0.60885
	Average	0.34396	0.36141	0.01746	0.34249
Panel B: 1/2/2001-1/2/2006	Min	0.11025	0.11625	-0.00018	0.06203
	Max	0.54778	0.61222	0.12420	0.57325
	Average	0.29828	0.32587	0.02759	0.36126
Panel C: 2/2/2006-1/2/2011	Min	0.16629	0.18267	0.00049	0.12017
	Max	0.65708	0.66163	0.03488	0.68647
	Average	0.41702	0.42687	0.00985	0.36491
Panel D: 2/2/2011-1/2/2017	Min	0.13981	0.17509	-0.00011	0.16574
	Max	0.64329	0.65223	0.08520	0.52142
	Average	0.37636	0.39039	0.01403	0.31714

Source: Own analysis

We can see from Table 3.5 that the R^2 values of the five–factor model are not much larger than those of three–factor model. In very rare cases, the improvement in R^2, which is given by $R^2_{5f} - R^2_{3f}$ exceeds 0.100. The vast majority of the cases demonstrates improvement of only between 0.000 to 0.124 with maximal average of 0.028, which is seen in Panel B. During the examined period the average improvement stands only on 0.017 while during other subperiods it is much lower. In Panel A such improvement does not exceed 0.071 and in Panel C such improvement does not exceed 0.035. In Panels B and D minimal values for the improvement are negative, suggesting that in some cases five–factor does not add to the coefficient of explanation, but reduces it. Moreover, comparing the values of three/five–factor models with those of the *CAPM* for the Israeli market we will see much similarity. The maximal and average values are close in both markets. The minimal values of the *CAPM* are much lower. This is because 3 companies from the Israeli market (BAZEQ, ISRAMCO and TEVA) that exhibit permanently low results, which in turn indicates their low financial position. The improvement of explanatory power of five–factor model is not impressive relatively to the three–factor model. In general, the average explanatory power of the five–factor model falls between 0.326 to 0.427 which can be comparable to those of the technical variables results in some cases. The average explanatory power of the three–factor model falls between 0.298 to 0.417. The absolute values for both models vary from about 0.110 to 0.650.

Obviously, the difference between three–factor and five–factor models is addition of two more factors. However, this action affects the coefficients of already existed *SMB* and *HML* as reflected in Table 3.5. The influence appears in 2 following forms.

First, *SMB* and *HML* may change their signs to opposite. It takes place for the *SMB* in 7 cases in Panel A (14% of all cases) and in 8 cases from Panel B (16%). For the *HML* in 11 cases in Panel B (22%); in 3 cases from Panel C, both *SMB* and *HML* change their sign (6% of all cases); in 4 cases from Panel D *SMB* changes the sign while *HML* changes the sign in 9 cases (8% and 18% respectively).

Second, *SMB* and *HML* may change their significance when mostly the *SBM* coefficients become insignificant while *HML* coefficients turn to be significant. The changes take place for Panel A in 12 cases for the *SMB* and in 13 cases for the *HML* (24% and 26% of all cases). Panel B demonstrates changes in 23 cases for the *SMB* and in 18 cases for the *HML* (46% and 36%); in 9 cases from Panel C the *SMB* changes its significance and the *HML* changes its significance in 13 cases (18% and 26%); from Panel D, *SMB* changes the significance in 22 cases while *HML* changes the significance only in 15 cases (44% and 30% respectively).

The underlined above points indicate strong instability of the coefficients of the Fama–French models and may convince one that the benefits of Fama–French models are not so good as it is claimed. However, this postulate is not accurate. The factor models are found to be suitable with monthly average returns instead of daily returns, used in the thesis. More than 30 years are involved in the samples of Fama and French (1993, 2012, 2015) though in the thesis it is 16 trading years. In addition, Fama and French (1993, 2012, 2015) apply their models to aggregate market returns but not to single stock returns, as it appears in the study.

Table 3.6 Distribution of the coefficients of three and five factor models by pattern and significance

3-factor: $RIRF_t^i = \alpha + \beta_1 (Mkt - R^f)_t + \beta_2 SMB_t + \beta_3 HML_t + \varepsilon_t$
5-factor: $RIRF_t^i = \alpha + \beta_1 (Mkt - R^f)_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 RMW_t + \beta_5 CMA_t + \varepsilon_t$

		3-factor			5-factor				
		$(Mkt - R^f)_t$	SMB_t	HML_t	$(Mkt - R^f)_t$	SMB_t	HML_t	RMW_t	CMA_t
Panel A: 1/2/2001-1/2/2017	Negative		10	44		17	44	34	23
	Positive	50	40	6	50	33	6	16	27
	Sign change	N/A	N/A	N/A		7		N/A	N/A
	1%	50	37	42	50	33	45	43	30
	5%		3	2		6	1	1	6
	10%		3	2		4	1	1	3
	10%>		7	4		7	3	5	11
	Signif. Down	N/A	N/A	N/A		7	5	N/A	N/A
	Signif. Up	N/A	N/A	N/A		5	8	N/A	N/A
Panel B: 1/2/2001-1/2/2006	Negative		10	33		15	36	30	26
	Positive	50	40	17	50	35	14	20	24
	Sign change	N/A	N/A	N/A		8	11	N/A	N/A
	1%	50	32	30	50	23	23	34	19
	5%		4	2		4	6	1	7
	10%		2	2		4	2	3	2
	10%>		12	16		19	19	12	22
	Signif. Down	N/A	N/A	N/A		16	12	N/A	N/A
	Signif. Up	N/A	N/A	N/A		7	6	N/A	N/A
Panel C: 2/2/2006-1/2/2011	Negative		9	45		11	44	37	20
	Positive	50	41	5	50	39	6	13	30
	Sign change	N/A	N/A	N/A		3	3	N/A	N/A
	1%	50	32	29	50	32	36	33	29
	5%		4	3		1	3	2	7
	10%			4		3	1	3	2
	10%>		14	14		14	10	12	12
	Signif. Down	N/A	N/A	N/A		6	4	N/A	N/A
	Signif. Up	N/A	N/A	N/A		3	9	N/A	N/A
Panel D: 2/2/2011-1/2/2017	Negative		24	43		24	44	21	20
	Positive	50	26	7	50	26	6	29	30
	Sign change	N/A	N/A	N/A		4	9	N/A	N/A
	1%	50	17	38	50	22	25	33	17
	5%		9	1		7	9	3	7
	10%		3			3	1	2	5
	10%>		21	11		18	15	12	21
	Signif. Down	N/A	N/A	N/A		13	9	N/A	N/A
	Signif. Up	N/A	N/A	N/A		9	6	N/A	N/A

Source: Own analysis

For short periods and daily returns the performance of the Fama–French models is obviously not impressive. However, rational–based investor will apply such models also to daily returns since to my knowledge no other fundamental model suitable with daily return ever exists, probably except the *CAPM*. The points sighted above keep a place to a better improvement which may come from the side of the Unified Capital Asset Pricing Model.

3.3.2. Results for the models, derived from PCA

Tables 3.7, 3.8a, 3.8b and 3.9 as well as Table 3.8c in the Annex summarize and compare results for technical, sentiment and unified model regressions, derived from the *PCA* according to the next equations respectively:

$$RIRF_t^i = \alpha + \beta_1 TA1_t + \beta_2 TA2_t + \beta_3 TA3_t + (\beta_4 TA4_t) + \varepsilon_t, \tag{3.16}$$

and

$$RIRF_t^i = \alpha + \beta_1 SI1_t + (\beta_2 SI2_t) + \varepsilon_t, \tag{3.17}$$

and

$$RIRF_t^i = \alpha + \beta_1 ALL1_t + \beta_2 ALL2_t + \beta_3 ALL3_t + \beta_4 ALL4_t + (\beta_5 ALL5_t) + \varepsilon_t. \tag{3.18}$$

The *PCA* parsimoniously incorporate information from a large number of potential predictors in a predictive regression. It reduces the number of predictors by converting them through orthogonal transformation to linearly uncorrelated principal components. As a result, potential multicollinearity is eliminated. However, every single original variable loses its importance, the *PCA* does not allow to see a direct influence of a single variable. Fortunately, in this study there is no real importance to the influence of a single variable but to their aggregative influence. The only necessary condition is the relevance of chosen original variables. Here, all the variables are chosen due to existing literature and they are proved to be relevant.

Table 3.7 demonstrates a percentage of original variation explained by retained principal components for both markets.

For both markets:

- retained components from technical indicators are able to explain about 66% to 75% with the average of about 70% of the original variation;
- components from sentiment indicators are able to explain about 35% to 75% with the average of about 65.5% of the original variation;
- components from the unified indicators are able to explain about 62.5% to 74.6% with

the average of about 69% of the original variation.

Table 3.7 Original variation explained after PCA

		US			Israel		
		Technical	Sentiment	Unified	Technical	Sentiment	Unified
Panel A: 1/2/2001-1/2/2017	Min	0.67150	0.46760	0.65670	0.71890	0.46590	0.65710
	Max	0.74280	0.74160	0.73300	0.74020	0.74840	0.73280
	Average	0.70119	0.65552	0.69178	0.73072	0.66341	0.69298
Panel B: 1/2/2001-1/2/2006	Min	0.66220	0.47900	0.62500	0.68200	0.44600	0.62840
	Max	0.75080	0.78630	0.74560	0.74570	0.74030	0.72890
	Average	0.70095	0.66445	0.68683	0.72181	0.57645	0.67274
Panel C: 2/2/2006-1/2/2011	Min	0.64710	0.46720	0.63970	0.67560	0.65780	0.65860
	Max	0.74430	0.73020	0.75020	0.74420	0.81000	0.74740
	Average	0.70365	0.66087	0.69249	0.72313	0.69254	0.69573
Panel D: 2/2/2011-1/2/2017	Min	0.66830	0.35260	0.63760	0.67510	0.46250	0.65720
	Max	0.74010	0.75160	0.73690	0.74400	0.72680	0.73150
	Average	0.70560	0.65159	0.69732	0.71862	0.64959	0.70776

Source: Own analysis

In the case of sentiment indicators, the minimum is relatively low. This is because of low number of predictors retained from the *PCA*. In the case of sentiment one or two predictors are retained where, naturally, one predictor has the ability to explain much lower part of variation which is reflected in low results for sentiment minimum. Despite this fact, the results of all models are very comparable. Moreover, the US as well as the Israeli markets demonstrate very similar results during all periods. It seems like very large part of the original variation is explained. However, there is no direct projection of high rate of explained variation on high explanatory power of the components within regressions.

Examining Tables 3.8a and 3.8b, Panels A to D, it is possible to figure out that the *PCA* for the technical indicators allows 3 to 4 components while for the sentiment indicators it allows 1 to 2 components. The *PCA* for the unified indicators allows 4 to 5 components. The situation is identical for both markets, suggesting it can probably appear on international level, though this phenomenon should be investigated deeply further.

The principal components for all indicators exhibit very stable coefficient pattern:

- for the technical indicators 1st component is positive, 2nd and 3rd components are negative and the 4th component is mostly negative;
- for the sentiment indicators 1st component is positive while 2nd component can be positive or negative;
- for the unified indicators 1st component is positive, 2nd to 3rd components are negative and the 4th component is mostly negative while 5th component can be positive or

negative.

However, it is difficult to conclude about the existence of any pattern for sentiment model since a number of coefficients is very small. The 1st predictor seems to be always significant for all cases and markets. The 2nd predictor, as the last for the sentiment model, has a very similar behavior to the last predictor of the technical model, i.e. it can be negative, positive or absent. Though if there were more sentiment variables and hence more *PCA* predictors, it is likely that a pattern would be more clear and vivid. However, once again, those patterns for all models are the same for both markets.

Table 3.8a, Panels A to D, summarizes results for technical, sentiment and unified components obtained from the US market. The results demonstrate that the vast majority of technical predictors is significant on the level of 1%. It is found that the 4th component rarely can be insignificant or significant at the level of 5% and only several cases of insignificance are recorded for the 3rd component. For the whole examined period (which is in Panel A) there is only 1 insignificant technical predictor (0.6% of all cases); only 3 predictors are insignificant in Panel B (1.8%); only 7 predictors are insignificant in Panel C (4.2%) and only 4 predictors have significance lower than 10% in Panel D (2.3%).

Regarding to the sentiment coefficients, there is only 1 insignificant predictor, PCAR in Panel D. Other coefficients are always significant at the level of 1%.

The unified model claims that non–fundamental component is composed of technical and sentiment both together, but not separately. Hence, the unified *PCA* predictors are composed of technical and sentiment variables, which are integrated predictors.

The results show that **vast majority of unified predictors are significant at the level of 1%**. For both markets, 3rd and 4th components can be insignificant in the same order, though 5th component can be insignificant more frequently. Mostly, if there is any insignificant coefficient for technical or sentiment predictors, this almost automatically leads to insignificance of 5th unified predictor for the same company. For the US market there are only 11 insignificant unified predictors in Panel A (5.1% of all cases); in Panel B 14 predictors have lower level of significance than 10%; only 13 predictors in Panel C (6.1%) and only 20 predictors in Panel D (9.1%) are insignificant. The significance of unified components is much lower than those of technical or sentiment, however, the insignificant coefficients have very negligible values and can be dropped without harming the explanatory power. The purpose of the thesis is to create some universal platform, hence all the coefficients for all models were obtained on the same basis and were not dropped in further calculations.

Table 3.8a Distribution of indicators, derived from PCA by pattern and significance on the US market

Technical: $RIRF_t^i = \alpha + \beta_1 TA1_t + \beta_2 TA2_t + \beta_3 TA3_t + (\beta_4 TA4_t) + \varepsilon_t$
Sentiment: $RIRF_t^i = \alpha + \beta_1 SI1_t + (\beta_2 SI2_t) + \varepsilon_t$
Unified: $RIRF_t^i = \alpha + \beta_1 ALL1_t + \beta_2 ALL2_t + \beta_3 ALL3_t + \beta_4 ALL4_t + (\beta_5 ALL5_t) + \varepsilon_t$

		Technical indicators				Sentiment indicators		Unified indicators				
		$TA1_t$	$TA2_t$	$TA3_t$	$(TA4_t)$	$SI1_t$	$(SI2_t)$	$ALL1_t$	$ALL2_t$	$ALL3_t$	$ALL4_t$	$(ALL5_t)$
Panel A: 1/2/2001-1/2/2017	Negative		50	50	18		24		50	46	23	7
	Positive	50			1	50	19	50		4	27	8
	NONE				31		7					35
US market - 50	1%	50	50	50	16	50	43	50	50	50	35	8
securities	5%				2						5	1
	10%										3	2
	10%>				1						7	4
Panel B: 1/2/2001-1/2/2006	Negative		50	50	18		23		50	47	25	7
	Positive	50				50	19	50		3	20	5
	NONE				32		8				5	38
US market - 50	1%	50	50	49	15	50	41	50	50	46	30	11
securities	5%				1		1				5	
	10%										1	
	10%>			1	2					4	9	1
Panel C: 2/2/2006-1/2/2011	Negative		50	50	15		18		50	40	28	6
	Positive	50			2	50	24	50		10	20	6
	NONE				33		8				2	38
US market - 50	1%	50	50	45	13	50	42	50	50	47	36	6
securities	5%			2							5	1
	10%										2	
	10%>			3	4					3	5	5
Panel D: 2/2/2011-1/2/2017	Negative		50	50	19		14		50	46	28	9
	Positive	50			3	50	27	50		4	22	10
	NONE				28		9					31
US market - 50	1%	50	50	49	18	50	40	50	50	49	32	11
securities	5%				1						1	
	10%									1	2	3
	10%>			1	3		1				15	5

Source: Own analysis

Table 3.8b, Panels A to D, summarizes results for technical, sentiment and unified components obtained from the Israeli market. From the results it is possible to learn that **vast majority of technical predictors are significant on the level of 1%**, but there is a much higher number of significant coefficients for the US market than for the Israeli one. In the Israeli case the 3rd component can be insignificant more frequently. For the Israeli market there are only 3 insignificant technical predictors in Panel A (5.3% of all cases); no insignificant predictors in Panel B, only 4 predictors in Panel C (7.4%) and only 1 insignificant predictor in Panel D (1.9%). Naturally, the Israeli market contains smaller sample, hence the percentage is higher, though even in this situation it is very comparable to the results for the US market. Exactly as on the

US market, the insignificant components have very low value coefficients, therefore its contribution to the overall explanation can be dropped without harm to it.

Table 3.8b Distribution of indicators, derived from PCA by pattern and significance on the Israeli market

Technical: $RIRF_t^i = \alpha + \beta_1 TA1_t + \beta_2 TA2_t + \beta_3 TA3_t + (\beta_4 TA4_t) + \varepsilon_t$

Sentiment: $RIRF_t^i = \alpha + \beta_1 SI1_t + (\beta_2 SI2_t) + \varepsilon_t$

Unified: $RIRF_t^i = \alpha + \beta_1 ALL1_t + \beta_2 ALL2_t + \beta_3 ALL3_t + \beta_4 ALL4_t + (\beta_5 ALL5_t) + \varepsilon_t$

		Technical indicators				Sentiment indicators		Unified indicators				
		$TA1_t$	$TA2_t$	$TA3_t$	$(TA4_t)$	$SI1_t$	$(SI2_t)$	$ALL1_t$	$ALL2_t$	$ALL3_t$	$ALL4_t$	$(ALL5_t)$
Panel A: 1/2/2001-1/2/2017	negative		14	13	14		9		14	12	12	5
	positive	14		1		14	4	14		2	2	4
	NONE						1					5
Israeli market - 14 securities	1%	14	14	11	14	14	13	14	14	12	12	5
	5%									1	1	
	10%											
	10%>			3						1	1	4
Panel B: 1/2/2001-1/2/2006	negative		14	13	11		1		14	13	13	
	positive	14		1		14	5	14		1	1	1
	NONE				3		8					13
Israeli market - 14 securities	1%	14	14	13	10	14	5	14	14	14	13	1
	5%				1							
	10%			1			1					
	10%>										1	
Panel C: 2/2/2006-1/2/2011	negative		14	14	12		9		14	11	10	2
	positive	14				14	5	14		3	4	5
	NONE				2							7
Israeli market - 14 securities	1%	14	14	11	11	14	14	14	14	12	9	2
	5%										1	
	10%											
	10%>			3	1					2	4	5
Panel D: 2/2/2011-1/2/2017	negative		14	14	12		8		14	11	12	8
	positive	14				14	4	14		3	2	4
	NONE				2		2					2
Israeli market - 14 securities	1%	14	14	10	7	14	12	14	14	13	13	5
	5%			2	2							2
	10%			2	2					1		1
	10%>				1						1	4

Source: Own analysis

The sentiment predictors are all significant at 1% though one predictor is significant at the level of 10%. Such situation is very similar to the one of the US market.

The unified model demonstrates lower performance. There are only 6 unified predictors, which are insignificant in Panel A (8% of all cases); only 1 insignificant predictor in Panel B (1.8%); 11 insignificant predictors in Panel C (17%) and only 5 insignificant predictors in Panel D (7.4%). Also here, as in the US, the insignificant coefficients have very negligible values and can be dropped without harming the explanatory power. Since the Israeli market contains much smaller sample, the percentage is much higher and it is difficult to correlate with the results of the US market.

Table 3.9 summarizes the values of R^2 for all models, derived from *PCA* obtained on the US and Israeli markets.

Table 3.9 $\mathbf{R^2}$ values of the models, derived from PCA

			R_{TA}^2	R_{SI}^2	R_{ALL}^2	$R_{ALL}^2 - R_{TA}^2$	$R_{ALL}^2 - R_{SI}^2$
Panel A: 1/2/2001-1/2/2017	US market	Min	0.21287	0.10580	0.37463	0.12441	0.14537
		Max	0.30259	0.33254	0.49878	0.24209	0.38302
		Average	0.26537	0.19377	0.45478	0.18941	0.26101
	Israeli market	Min	0.14043	0.12270	0.29228	0.07784	0.11870
		Max	0.32405	0.26201	0.49359	0.19223	0.34852
		Average	0.27002	0.18518	0.43235	0.16233	0.24717
Panel B: 1/2/2001-1/2/2006	US market	Min	0.22289	0.07090	0.35317	0.07258	0.10482
		Max	0.32318	0.39201	0.55651	0.30689	0.37587
		Average	0.27640	0.23394	0.47242	0.19602	0.23848
	Israeli market	min	0.14685	0.07135	0.23922	0.06422	0.10448
		max	0.36123	0.38358	0.51431	0.21266	0.37551
		Average	0.29040	0.17867	0.44090	0.15050	0.26223
Panel C: 2/2/2006-1/2/2011	US market	Min	0.22074	0.10328	0.38432	0.14423	0.08534
		Max	0.31612	0.44614	0.55164	0.31807	0.39682
		Average	0.27335	0.23987	0.47780	0.20445	0.23792
	Israeli market	Min	0.10344	0.12986	0.30820	0.08020	0.10815
		Max	0.33998	0.25632	0.51387	0.21244	0.32573
		Average	0.27211	0.20689	0.43993	0.16782	0.23304
Panel D: 2/2/2011-1/2/2017	US market	Min	0.20019	0.07987	0.39410	0.10943	0.09799
		Max	0.34406	0.49210	0.59009	0.38990	0.44619
		Average	0.28987	0.24019	0.49975	0.20988	0.25957
	Israeli market	Min	0.24581	0.10986	0.35579	0.07918	0.19938
		Max	0.32003	0.29305	0.52105	0.27524	0.38674
		Average	0.30047	0.20761	0.47561	0.17514	0.26800

Source: Own analysis

The R^2 of technical predictors for the US market varies from around 0.200 to 0.344 with the average of 0.276. For the Israeli market it varies from around 0.100 to 0.361 with the average of 0.270. The extreme measures of about 0.100 or 0.146 are due to 3 companies with permanently low performance. However, it still comparable to the results of the US market. Recalling the results from Table 3.4 it is possible to see that R^2 is not much different, meaning during the *PCA* only very little part of the original information was lost.

The sentiment component may have 1 or 2 predictors. One predictor demonstrates low results for explanatory power relatively to those of 2 predictors, even despite the fact that some results of R^2 for 2 predictors can be very low, less than 20%. Hence, one *PCA* predictor cannot be sufficient and larger number of good sentimental variables should be involved in the *PCA* process. The lack of the number of qualitive and relevant sentimental variables is felt for both markets. In general, for the US market R^2 of sentiment predictors varies from approximately 0.070 to 0.492 with the overall average, including 1 predictor case, of about 0.240. In some cases, even within 2 predictors, the R^2 may fall below 10%. For the Israeli market R^2 of sentiment predictors may vary from around 0.071 to 0.384 with overall average, including 1 predictor, of about 0.207. Also, on the Israeli market there are some cases, even within 2 predictors, when the R^2 may fall below 10% or to be close to it. The results are still being low, which once again suggests to search for more sentiment variables. Comparing the results of explanatory power for the sentiment predictors in Table 3.9 with Table 3.4, it is possible to see a dramatic drop in the values of R^2 after the *PCA*. The difference is due to multicollinearity among the original sentiment variables. The overall picture of both markets is similar, suggesting again that such phenomenon may take place on the international a level.

As for the R^2 of the unified predictors, the values increased dramatically in comparison with technical or sentiment models and in the vast majority of all cases it is greater than 45%. The contribution to explanatory power of the unified model to technical and sentiment models is presented by $R^2_{ALL} - R^2_{TA}$ and $R^2_{ALL} - R^2_{SI}$ respectively. For the US market the unified model adds to explanation from around 0.073 to 0.390 with the average of about 0.200 in the case of technical model and from 0.085 to 0.446 with the average of about 0.260 in the case of sentiment model. For the Israeli market the unified model records have less impressive contribution to the technical model but similar contribution to the sentiment model, which stays close to those of the US market. It is from about 0.060 to 0.275 with the average of around 0.170 in the case of technical model and from about 0.100 to 0.387 with the average of around 0.260 in the case of sentiment model. In absolute terms, for the US market, the values of R^2 vary from 0.353 to 0.590 with the average of about 0.480. While in the case of Israeli market it varies between around 0.239 to 0.521 with the average of about 0.450. The results for the Israeli market are somewhat lower than those of the US market but are still close enough.

Interesting phenomenon regarding the intercepts of all regressions appears. The intercepts can be negative or positive, but mostly insignificant or significant at the level of 10% (close to be insignificant) with low values. Though the unified model increases the significance of the intercepts in general, extremely rarely it can be significant at the level of 1%. However, the phenomenon refers to the issue of absolute invariableness of the intercepts per a company. For a single company, the intercept is absolutely the same during technical, sentiment and unified regressions. Such phenomenon remains non–understandable and need to be investigated deeper.

3.3.3. Comparison with normative and behavioral models

Tables 3.10a, 3.10b, 3.11a, 3.11b, 3.12 and 3.13 as well as Tables 3.11c, 3.11d and 3.11e in the Annex summarize results for integrated regressions of technical, sentiment and unified models with relevant fundamental component for the US and Israeli markets respectively. The results are obtained due to equations (3.6) to (3.11).

Table 3.10a demonstrates the values of R^2 and its contribution to the relevant fundamental component which is given by $R_{TA}^2 - R_{5f}^2$, $R_{SI}^2 - R_{5f}^2$ and $R_{ALL}^2 - R_{5f/CAPM}^2$ for technical, sentiment and unified models respectively.

Table 3.10a $\mathbf{R^2}$ values of the integrated with fundamental components models

			R_{TA}^2	R_{SI}^2	R_{ALL}^2	$R_{TA}^2 - R_{5f}^2$	$R_{SI}^2 - R_{5f}^2$	$R_{ALL}^2 - R_{5f/CAPM}^2$
Panel A: 1/2/2001-1/2/2017	US market	Min	0.33690	0.32212	0.45318	0.04975	0.02117	0.10937
		Max	0.65892	0.63034	0.71854	0.16656	0.20763	0.31579
		Average	0.47210	0.45690	0.58216	0.11069	0.09548	0.22074
	Israeli market	Min	0.21304	0.18746	0.32613	0.06809	0.02452	0.12397
		Max	0.67694	0.63956	0.73271	0.17714	0.17157	0.29937
		Average	0.46496	0.43228	0.56058	0.12247	0.08979	0.21811
Panel B: 1/2/2001-1/2/2006	US market	Min	0.30781	0.27313	0.45370	0.03202	0.02671	0.11625
		Max	0.65448	0.67157	0.71022	0.22893	0.28915	0.61222
		Average	0.45198	0.45686	0.58032	0.12611	0.13099	0.32587
	Israeli market	Min	0.17785	0.11498	0.25533	0.04363	0.02485	0.06971
		Max	0.64602	0.68010	0.70909	0.16612	0.21706	0.32741
		Average	0.47528	0.43506	0.56463	0.11402	0.07380	0.20337
Panel C: 2/2/2006-1/2/2011	US market	Min	0.35199	0.33307	0.54493	0.03927	0.01846	0.08805
		Max	0.70090	0.71611	0.74968	0.16932	0.28651	0.38845
		Average	0.53177	0.53726	0.64151	0.10490	0.11039	0.21464
	Israeli market	Min	0.17786	0.24594	0.33849	0.04414	0.03385	0.08258
		Max	0.73061	0.72032	0.76905	0.24400	0.17492	0.35520
		Average	0.48837	0.45699	0.58286	0.12345	0.09207	0.21795
Panel D: 2/2/2011-1/2/2017	US market	Min	0.28411	0.33850	0.51910	0.04748	0.01872	0.08647
		Max	0.69971	0.69731	0.73870	0.16538	0.42466	0.45828
		Average	0.50219	0.51326	0.62201	0.11180	0.12288	0.23162
	Israeli market	Min	0.38766	0.30154	0.49613	0.09879	0.02669	0.17530
		Max	0.62919	0.58603	0.70841	0.22192	0.17323	0.31598
		Average	0.47828	0.43217	0.59096	0.16114	0.11504	0.18479

Source: Own analysis

After integration of the fundamental components all models demonstrate higher values of $\boldsymbol{R^2}$. On the US market the technical model contributes to explanation from around 0.050 to 0.167 during the whole examined period, adding about 0.111 on average. During other subperiods the results are similar, but in some extreme cases, the contribution value may rise to 0.229 or fall to 0.032. On the Israeli market those values vary between 0.068 and 0.177

during the whole examined period, adding about 0.122 to explanation on average. During other subperiods such results are similar or better; for example, Panel C recorded the highest contribution value of 0.244. However extremely low as well as extremely high values are rare and due to specific internal characteristics of those companies.

In absolute terms the technical model in the US is successful in explanation of between 0.308 and 0.659 for Panels A and B with the average of 0.452–0.472. Panels C and D demonstrate a little higher ability which is between 0.280–0.350 to 0.701 with the average of 0.502–0.532. In Israel these values mostly fall between 0.320 and 0.610 with some extremal exception of lowest 0.178 or highest 0.770 from the other side. The average values fall in a range of 0.465–0.488.

Regarding to the sentiment model, in general the results are comparable to those of technical model. On the US market the sentiment contribution varies between about 0.021 and 0.208 with the average of 0.095 for the whole examined period. However, during other subperiods it may vary from 0.018 to 0.290 with the average of 0.110 to 0.131 when the biggest contribution recorded for VRTX and stands on 0.426, but this extremely high result is the absolute exception. The second highest result for this period is 0.253 which is absolutely normal. In Israel for the whole examined period, the contribution may vary between 0.025 to 0.172 with the average of 0.090. Other subperiods demonstrate similar results, with highest contribution of 0.217. The results for both markets are similar to those of technical model.

In absolute terms the values of R^2 for the US market may vary between 0.273 to 0.716 with the average of about 0.457–0.537 within all panels. However, for the whole examined period the range is 0.322 to 0.630 with the average of around 0.457. Regarding to the Israeli market, the overall range is about 0.115 to 0.720 with the average of 0.432–0.457. However, those values are extremal for Israel due to a small number of companies involved in the research. The normal range seems to be smaller, about 0.245 to 0.590 within the same average. For the whole examined period the range is 0.187 to 0.640 with the average of 0.432.

Obviously, **technical and sentiment models have the ability to add to the explanation of fundamental models, however the highest contribution to the explanation is recorded by the unified model**. For the US market such contribution may vary between approximately 0.109 to 0.316 for the whole examined period with the average of about 0.221. During other subperiods the results are even better, when it can easily exceed 0.300 and reach exceptional maximum of 0.612. For the examined period in Israel such contribution may add to fundamental model's explanation between around 0.124 to 0.299 with the average of 0.218. Other periods exhibit somewhat less impressive results with the highest record of 0.355, which in general is close to the results on the US market. **The contribution of the unified model is twice bigger than those of the technical model and even bigger more in the case of sentiment model. This situation can be traced throughout all subperiods**.

Comparing the contribution of all models, the unified model transcends technical one with between 0.060 to 0.184 and 0.110 on average for the whole examined period in the US, while its superiority on the sentiment model varies between 0.062 and 0.220 with 0.125 on average, which is once again comparable to the performance of the technical model. During other subperiods this parameter falls to 0.040 and may raise to 0.350, in Panel D with the average contribution of 0.110. Regarding to the Israeli market, the superiority on the technical model is about 0.056 to 0.156, adding 0.096 on average for the same period. However, this superiority may fall to 0.026 in Panel B or raise only to 0.192 in Panel D. The superiority on the sentiment model may vary from 0.083 to 0.185 with the average of 0.128.

In absolute terms the values of R^2 for the unified model on the US market may vary between 0.453 to 0.720 with the average of 0.580 for Panels A and B. In other panels, such values vary from 0.520 to 0.750 with the average of more than 0.620. Regarding the Israeli market these values vary from 0.255 to 0.730 with the average of 0.560, though the normal range is about 0.430 to 0.700, and 0.340 to 0.770 with the average of 0.580 for the parallel periods. The results of the US and Israeli markets look very similar once again.

Table 3.10b demonstrates the distribution of R^2 values per deciles regarding every examined model. Here, R^2 values of technical and sentiment models are able to exceed 0.4. However, regarding to the 0.5 threshold which applies to the explanatory performance hypothesis (H2), both models exhibit low results. During the whole examined period in the US only 17 out of 50 within the technical model pass the threshold and 18 in the case of the sentiment model, which is much less than 50% of all cases. As about R^2 value of 0.6 there is only one case recorded for the sentiment during this period. Similar situation in other subperiods except Panel C, where both technical and sentiment R^2 values exceed the 0.5 threshold more than in 50% of all cases. In Israel the situation is quite similar. During the whole examined period only 6 out of 14 within the technical model and 5 in the case of sentiment model are able to exceed 0.5, which is also much less than 50% of all cases. Regarding other subperiods Panels C and B exhibit very close results for both models which is the best achievement for both but still be under 50% of all cases or equal to it. When the threshold increases, the results for both models permanently decline. However, 2 cases of more than 0.7 are recorded in Panel C — one for each model.

Table 3.10b Distribution of R^2 values of the integrated models per deciles

			$R^2 > 0.4$	$R^2 > 0.5$	$R^2 > 0.55$	$R^2 > 0.6$	$R^2 > 0.7$
Panel A: 1/2/2001-1/2/2017	US market – 50 securities	Technical	42	17	10	2	
		Sentiment	35	18	8	1	
		Unified	50	46	33	22	1
	Israeli market – 14 securities	Technical	9	6	5	2	
		Sentiment	8	5	2	2	
		Unified	12	11	8	6	2
Panel B: 1/2/2001-1/2/2006	US market – 50 securities	Technical	37	12	6	4	
		Sentiment	35	15	10	5	
		Unified	50	45	35	20	2
	Israeli market – 14 securities	Technical	9	7	6	4	
		Sentiment	8	6	5	1	
		Unified	12	10	9	7	2
Panel C: 2/2/2006-1/2/2011	US market – 50 securities	Technical	49	33	21	12	1
		Sentiment	47	37	19	10	1
		Unified	50	50	49	39	5
	Israeli market – 14 securities	Technical	9	7	6	3	1
		Sentiment	8	6	4	2	1
		Unified	13	11	8	7	2
Panel D: 2/2/2011-1/2/2017	US market – 50 securities	Technical	48	21	14	6	
		Sentiment	45	28	16	7	
		Unified	50	50	44	31	6
	Israeli market – 14 securities	Technical	13	4	3	2	
		Sentiment	8	4	1		
		Unified	14	13	12	4	1

Source: Own analysis

Almost every integrated regression for the unified model exceeds 0.5 of R^2: In Panels A only 4 companies out of 50 did not exceed 0.5 (8%); in Panels B only 5 companies out of 50 did not pass 0.5 (10%) and 100% exceeded 0.5 value in Panels C and D. Moreover, in a lot of cases R^2 even exceeds 0.6: in Panels A and B it is 46%; in Panels C it is 86% and in Panel D it is 72% of cases. The exception are the companies that at the stage of direct variables check before the principal component analysis had low values of R^2. However, those which do not exceed 0.5 still have significant increase in R^2 value caused by the unified model. Also in Israel the unified model exceeds 0.5 of R^2: in Panels A to C only 3 companies out of 14 did not exceed it (21%) and none in Panel D. Thus, in most cases it exceeds 0.55, in Panel A it is in 57%; in Panels B and C it is 64% and in Panel D it is 86% cases. The exception are the companies with low starting values like BAZEQ, ISRAMCO or TEVA. Those 3 companies demonstrate permanently low results for all examined periods and for all applied models. The reason is their internal characteristics. When the results of sentiment and technical models are close to each other, the unified model demonstrates much better performance than technical or sentiment models separately for both markets.

Despite the different fundamental component applied in Israeli and US markets, the overall results of all models for both markets look very similar. One of possible reasons is the quality of fundamental components. Indeed, looking at the Table 3.5, Panels A to D more carefully, it is possible to figure out that for the five–factor model applied to the US market only one coefficient is **always** significant with very high value relatively to other coefficients and it is $(\boldsymbol{Mkt - R^f})_{\boldsymbol{t}}$. This parameter is associated with market return, which is closely related to the *CAPM*, applied to the Israeli market. Other coefficients of five–factor components are less stable and may be less important to overall explanatory contribution even despite the fact that some of them have coefficients with high value — though this finding should be researched deeper. Other possible reason is that technical, sentiment and unified models hold on the international level. Noticeably, the fundamental component applied to the Israeli market demonstrates absolutely the same behavior as the $(Mkt - R^f)_t$ on the US market: it has coefficients with high value and it is **always** significant at the 1% level.

Table 3.11a demonstrates results for coefficient distribution by significance and sign pattern on the US market after integration with relevant fundamental components. The bold text in the table reflects the changes in the number of the coefficients that raised their level of significance comparing to those of Table 3.8a.

After the integration with the fundamental components on the US market, the level of significance of the coefficients of the technical and the unified predictors derived from the *PCA* is increased. This increase is especially prominent among the coefficients of the unified model and such phenomenon is observed during all periods. Generally, this increase is obtained for last predictors that more frequently can be insignificant, turning to be significant after the integration. Regarding the sentiment, it is hard to conclude whether a change in its coefficients occurred, since only one sentiment coefficient is insignificant and other coefficients are found at the highest level of significance of 1%. Additionally, the lack of the number of the sentiment variables prevents to conclude regarding the significance level after the integration when more relevant variables could be added to the analysis. This indicates that a combination of fundamental components with the technical and the unified indicators (also potentially with the sentiment indicators) is successful and even wishful.

Table 3.11a Distribution of integrated predictors by pattern and significance on the US market

Technical: $RIRF_t^i = \alpha + \beta_1 TA1_t + \beta_2 TA2_t + \beta_3 TA3_t + (\beta_4 TA4_t) + \beta_5 RES_t^{MKT} + \beta_6 RES_t^{SMB} + \beta_7 RES_t^{HML} + \beta_8 RES_t^{RMW} + $
$+ \beta_9 RES_t^{CMA} + \varepsilon_t$

Sentiment: $RIRF_t^i = \alpha + \beta_1 SI1_t + (\beta_2 SI2_t) + \beta_3 RES_t^{MKT} + \beta_4 RES_t^{SMB} + \beta_5 RES_t^{HML} + \beta_6 RES_t^{RMW} + \beta_7 RES_t^{CMA} + \varepsilon_t$

Unified: $RIRF_t^i = \alpha + \beta_1 ALL1_t + \beta_2 ALL2_t + \beta_3 ALL3_t + \beta_4 ALL4_t + (\beta_5 ALL5_t) + \beta_6 RES_t^{MKT} + \beta_7 RES_t^{SMB} + \beta_8 RES_t^{HML} + $
$+ \beta_9 RES_t^{RMW} + \beta_{10} RES_t^{CMA} + \varepsilon_t$

The bold text reflects the changes in the number of the coefficients that raised their level of significance comparing to those of Table 3.8a.

		Technical indicators				Sentiment indicators		Integrated indicators				
		$TA1_t$	$TA2_t$	$TA3_t$	$(TA4_t)$	$SI1_t$	$(SI2_t)$	$ALL1_t$	$ALL2_t$	$ALL3_t$	$ALL4_t$	$(ALL5_t)$
Panel A: 1/2/2001-1/2/2017	Negative		50	50	18		24		50	46	23	7
	Positive	50			1	50	19	50		4	27	8
	NONE				31		7					35
US market - 50 securities	1%	50	50	50	**17**	50	43	50	50	50	35	8
	5%				**1**						**6**	1
	10%										**2**	2
	10%>				1						7	4
Panel B: 1/2/2001-1/2/2006	Negative		50	50	18		23		50	47	25	7
	Positive	50				50	19	50		3	20	5
	NONE				32		8				5	38
US market - 50 securities	1%	50	50	49	15	50	42	50	50	46	**31**	11
	5%				1						**4**	
	10%									**1**	1	
	10%>			1	2					**3**	9	1
Panel C: 2/2/2006-1/2/2011	Negative		50	50	15		18		50	40	28	6
	Positive	50			2	50	24	50		10	20	6
	NONE				33		8				2	38
US market - 50 securities	1%	50	50	**46**	13	50	42	50	50	47	**37**	**7**
	5%			**1**							**4**	
	10%										2	
	10%>			3	4					3	5	5
Panel D: 2/2/2011-1/2/2017	Negative		50	50	19		14		50	46	27	8
	Positive	50			3	50	28	50		4	23	11
	NONE				28		8					31
US market - 50 securities	1%	50	50	49	18	50	41	50	50	49	32	**15**
	5%				1						**4**	
	10%									1	**1**	
	10%>			1	3		1				**13**	**4**

Source: Own analysis

Similarly, Table 3.11b demonstrates results for the coefficient distribution by significance and sign pattern after the integration with relevant fundamental component in Israel. Absolutely identically to the US market, the Israeli market demonstrates the same trend where the integration of the fundamental component increases the significance of the coefficients for the technical and the unified predictors comparing to Table 3.8b. Also on the Israeli market the changes occur during all periods without to influence the sentiment indicators. Although the change of the level of significance exists, it is less prominent than those of the US market. The reason is relatively small sample. However, such situation allows concluding that with enlarging the sample of the Israeli market, the change in the significance will be more palpable.

Table 3.11b Distribution of integrated predictors by pattern and significance on the Israeli market

Technical: $RIRF_t^i = \alpha + \beta_1 TA1_t + \beta_2 TA2_t + \beta_3 TA3_t + (\beta_4 TA4_t) + \beta_5 RES_t^{MKT} + \beta_6 RES_t^{SMB} + \beta_7 RES_t^{HML} + \beta_8 RES_t^{RMW} +$
$+\beta_9 RES_t^{CMA} + \varepsilon_t$
Sentiment: $RIRF_t^i = \alpha + \beta_1 SI1_t + (\beta_2 SI2_t) + \beta_3 RES_t^{MKT} + \beta_4 RES_t^{SMB} + \beta_5 RES_t^{HML} + \beta_6 RES_t^{RMW} + \beta_7 RES_t^{CMA} + \varepsilon_t$
Unified: $RIRF_t^i = \alpha + \beta_1 ALL1_t + \beta_2 ALL2_t + \beta_3 ALL3_t + \beta_4 ALL4_t + (\beta_5 ALL5_t) + \beta_6 RES_t^{MKT} + \beta_7 RES_t^{SMB} + \beta_8 RES_t^{HML} +$
$+\beta_9 RES_t^{RMW} + \beta_{10} RES_t^{CMA} + \varepsilon_t$
The bold text reflects the changes in the number of the coefficients that raised their level of significance comparing to those of Table 3.8b.

		Technical indicators				Sentiment indicators		Integrated indicators				
		$TA1_t$	$TA2_t$	$TA3_t$	$(TA4_t)$	$SI1_t$	$(SI2_t)$	$ALL1_t$	$ALL2_t$	$ALL3_t$	$ALL4_t$	$(ALL5_t)$
Panel A: 1/2/2001-1/2/2017	Negative		14	13	14		9		14	11	12	5
	Positive	14		1		14	4	14		3	2	4
	NONE						1					5
Israeli market - 14 securities	1%	14	14	11	14	14	13	14	14	12	13	5
	5%									1	1	**1**
	10%											
	10%>			3						1		**3**
Panel B: 1/2/2001-1/2/2006	Negative		14	13	11		1		14	13	13	
	Positive	14		1		14	5	14		1	1	1
	NONE				3		8					13
Israeli market - 14 securities	1%	14	14	13	**11**	14	5	14	14	14	13	1
	5%											
	10%			1			1					
	10%>										1	
Panel C: 2/2/2006-1/2/2011	Negative		14	14	12		9		14	11	10	2
	Positive	14				14	5	14		3	4	5
	NONE				2							7
Israeli market - 14 securities	1%	14	14	11	11	14	14	14	14	12	**9**	2
	5%										1	
	10%									**1**	**1**	
	10%>			3	1					**1**	**3**	5
Panel D: 2/2/2011-1/2/2017	Negative		14	14	12		8		14	11	12	12
	Positive	14				14	4	14		3	2	
	NONE				2		2					2
Israeli market - 14 securities	1%	14	14	**14**	**10**	14	12	14	14	13	13	**6**
	5%				**1**							**1**
	10%									1		1
	10%>				1						1	4

Source: Own analysis

The integration of fundamental models left one question opened: what happens to the coefficients of the fundamental models after the integration? At the end of part 3.3.1 it was shown that some coefficients of five–factor model may change their signs and significance level as a result of adding 2 more factors. Indeed, after the integration of technical, sentiment and unified models with the fundamental components, the pattern and the level of significance of the fundamental factors coefficients are changed.

Table 3.12a demonstrates the results for the distribution of fundamental factor coefficients by pattern for all integrated models on the US market. Also the sign changes are shown. As about the Israeli market, there is only one fundamental component which **always** keeps its positive sign and significance at 1% level, hence is not represented in any table.

Table 3.12a Distribution of fundamental factor coefficients by pattern

		RES_t^{MKT}	RES_t^{SMB}	RES_t^{HML}	RES_t^{RMW}	RES_t^{CMA}
Panel A: 1/2/2001-1/2/2017	Negative		23	44	32	24
Technical	Positive	50	27	6	18	26
	Sign change		6		1	1
	Negative		20	45	34	22
Sentiment	Positive	50	30	5	16	28
	Sign change		2			1
	Negative		24	44	33	26
Unified	Positive	50	26	6	17	24
	Sign change		6		2	3
Panel B: 1/2/2001-1/2/2006	Negative		17	36	29	29
Technical	Positive	50	33	14	21	21
	Sign change		3	1	2	5
	Negative		19	40	31	28
Sentiment	Positive	50	31	10	19	22
	Sign change		4	3	3	1
	Negative		23	43	22	22
Unified	Positive	50	27	7	28	28
	Sign change		7	6	3	5
Panel C: 2/2/2006-1/2/2011	Negative		18	43	31	17
Technical	Positive	50	32	7	19	33
	Sign change		5	1	4	3
	Negative		11	44	33	19
Sentiment	Positive	50	39	6	17	31
	Sign change			1	1	1
	Negative		21	44	31	18
Unified	Positive	50	29	6	19	32
	Sign change		11		4	2
Panel D: 2/2/2011-1/2/2017	Negative		26	41	27	22
Technical	Positive	50	24	9	23	28
	Sign change		6	1	2	3
	Negative		27	45	25	19
Sentiment	Positive	50	23	5	15	31
	Sign change		3		3	2
	Negative		27	38	27	23
Unified	Positive	50	23	12	23	27
	Sign change		3	3	3	6

Source: Own analysis

Regarding to the coefficient pattern of the technical model, the original signs are kept in general when it turns to opposite for the *SMB* in 6 cases out of 50 in Panel A and D (12% of all cases); in 3 cases from Panels B (6%) and in 5 cases in Panel C (10%). For the *HML* there is only 1 case in all panels except Panel A, where no sign change is recorded. Those results are much better comparing to the results of adding 2 more factors to the three–factor model (Table 3.5). Other factors may also change their signs, as for example, *RMW* changes its sign only in 1 case from Panel A; in 2 cases from Panel B and D as well as in 4 cases in Panel C. Similarly, *CMA* changes its sign in 1 case in Panel A; in 5 cases from Panel B and in 3 cases from Panel C and D. There are only several cases of sign changes recorded for the fundamental components, suggesting higher stability of the coefficients.

The integration of sentiment model demonstrates similar results to those of technical one though the sign matching is a little more accurate. During the examined period *SMB* changes its sign in 2 cases; *CMA* only in 1 case while *HML* and *RMW* have no changes in its signs. In Panel B, *SMB* changes its sign in 4 cases, *HML* with *RMW* only in 3 cases while *CMA* only in 1 case. In Panel C, *SMB* has no changes of its signs while all other factors record only 1 case of a sign change. In Panel D, *SMB* and *RMW* change its sign in 6 cases; *CMA* only in 2 cases while *HML* sign is unchanged.

Regarding to the unified model the results for the integration of fundamental factors are little less impressive relatively to sentiment or technical models though the coefficient pattern looks similar. Within the whole examined period, *SMB* changes its sign in 6 cases (12% of all cases); *RMW* in 2 cases (4%) and *CMA* in 3 cases (6%). From Panel B for *SMB* in 7 cases the sign is changed, for *HML* in 6 cases, *RMW* changes the signs in cases and *CMA* changes the sign in cases. In Panel C, *SMB* changes the sign in 11 cases, *RMW* in 4 cases and *CMA* only in 2 cases. Regarding Panel D, *SMB* with *RMW* and *HML* change their signs in 3 cases while *CMA* changes its signs in 6 cases. The unified model includes technical and sentiment components inside, hence it inherits the influence of both though in different rate. From here, if some of fundamental components are positive/negative within integrated technical model but demonstrate opposite signs within integrated sentiment model, the unified model inherits the sign of most influential model which is natural, as for example *SMB* coefficient of DLTR or CTAS. Mostly, if a sign is the same within both technical and sentiment models, naturally the same sign is inherited by the unified model. However, in some rare cases a given coefficient can demonstrate the same sign in the technical and sentiment models while in the integrated unified model it changes its sign to the opposite (for example, BIIB, CTAS, HOLX and LRCX). The phenomenon exists during all periods. With it all inconsistence in coefficient pattern of fundamental factors takes place only among insignificant coefficients or with 10% significance.

Table 3.12b demonstrates results for the distribution of fundamental factor coefficients by significance for all integrated models in the US. The results for technical and sentiment models are very similar and comparable. Comparing the coefficients of technical integrated model with original five–factor model allows us to conclude that after the integration the significance pattern of five–factor model does not change dramatically. Mostly, the originally insignificant coefficients are still insignificant or significant at the level of 10% (close to be insignificant) after the integration, though original signs may change.

Table 3.12b Distribution of fundamental factor coefficients by significance

		RES_t^{MKT}	RES_t^{SMB}	RES_t^{HML}	RES_t^{RMW}	RES_t^{CMA}
Panel A: 1/2/2001-1/2/2017	1%	50	34	44	37	33
	5%		2	3	5	2
Technical	10%		1			3
	10%>		13	3	8	12
	1%	50	33	44	40	33
Sentiment	5%		3	2	3	3
	10%		3		1	1
	10%>		11	4	6	13
	1%	50	31	42	36	33
Unified	5%		7	4	4	4
	10%		1	1	2	
	10%>		11	3	8	13
Panel B: 1/2/2001-1/2/2006	1%	50	17	21	32	20
	5%		9	6	3	5
Technical	10%		1	4	2	1
	10%>		23	19	13	24
	1%	50	17	23	33	22
Sentiment	5%		4	6	1	4
	10%		4	4	1	1
	10%>		25	17	15	23
	1%	50	14	22	26	21
Unified	5%		8	4	5	1
	10%		4	6	1	5
	10%>		24	18	18	23
Panel C: 2/2/2006-1/2/2011	1%	50	25	33	26	24
	5%		10	5	6	10
Technical	10%			1	3	3
	10%>		15	11	15	13
	1%	50	27	31	30	25
Sentiment	5%		7	6	4	6
	10%		4	1	4	5
	10%>		12	12	12	14
	1%	50	23	31	21	22
Unified	5%		6	4	7	9
	10%		1	3	2	5
	10%>		20	12	20	14
Panel D: 2/2/2011-1/2/2017	1%	50	12	28	26	16
	5%		1	2	1	6
Technical	10%		5	3	3	6
	10%>		28	16	20	22
	1%	50	18	32	23	21
Sentiment	5%		4	4	2	5
	10%		2	2	3	4
	10%>		26	12	22	20
	1%	50	16	24	19	15
Unified	5%		2	7	5	8
	10%		2	2	5	2
	10%>		30	17	21	25

Source: Own analysis

However, in some cases original coefficients at higher level of significance like 5% or even 1% may reduce their significance and even turn to be insignificant after the integration, as in the examples of AKAM, LRCX, MYL or XRAY. With it the opposite situation is allowed, when the integration may increase the level of significance to originally insignificant coefficients, like in the examples of ADBE, BIIB, MAT or SYMC.

The significance is mostly inherited by the unified model from the technical or sentiment models similar to the coefficient pattern. Exactly as in the case of coefficient pattern, there are some coefficients that change their significance level only in the unified model[32]. During the examined period there are 6 cases of significance increase and 3 cases of its decrease as a result of the integration, so the unified model is a sort of improvement for the technical and sentiment models. For instance, AKAM, AMAZN, AAPL, PCLN, SYMC and WDC or EA, ROST and ORLY respectively. However, during the subperiods there are much less of such examples, when mostly the significance is dropped. In vast majority of the cases this phenomenon refers to the coefficient of the *SMB*. In addition, there are a lot of cases when the technical or sentimental coefficients, originally significant at level of 10%, lose their significance during the unified model regressions though the opposite also takes place, i.e. originally insignificant technical or sentimental parameters in some cases become significant at level of 10% in the unified model regressions. Hence, it is difficult to conclude if there is any improvement in the significance level or not since the level of 10% could be much close to insignificance.

The performance of the unified model looks good, though one may argue that integration of all components separately may bring a better performance than the unified model and hence creation of such model is not justified. This would mean that it is possible to integrate directly technical and sentiment predictors derived from the *PCA* with relevant fundamental components and by this way to achieve better results than those of the unified model. The alternative model works much like previous mechanism, i.e. in the Israeli case:

$$CAPM_t = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + \beta_n \sum_{n=1}^{N} TA_{n,t} + u_t \rightarrow RES_t^{TASI};$$

[32] Such statement refers only to cases where the change in the significance level is meaningful, i.e. at least 2 levels difference in Table 3.13. The change of only one level is not enough to determine either the change is meaningful.

$$RIRF_t^i = \alpha + \beta_1 RES_t^{TASI} + \beta_k \sum_{k=1}^{K} SI_{k,t} + \beta_n \sum_{n=1}^{N} TA_{n,t} + \varepsilon_t, \quad (3.18)$$

$$K = \{k_1, \dots, k_n | eigenvalue \geq 1\}; N = \{n_1, \dots, n_n | eigenvalue \geq 1\}.$$

In the case of the US market:

$$E(R_t^M) - R_t^f = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + \beta_n \sum_{n=1}^{N} TA_{n,t} + u_t \rightarrow RES_t^{MKT};$$

$$SMB_t = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + \beta_n \sum_{n=1}^{N} TA_{n,t} + u_t \rightarrow RES_t^{SMB};$$

$$HML_t = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + \beta_n \sum_{n=1}^{N} TA_{n,t} + u_t \rightarrow RES_t^{HML};$$

$$RMW_t = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + \beta_n \sum_{n=1}^{N} TA_{n,t} + u_t \rightarrow RES_t^{RMW};$$

$$CMA_t = \alpha + \beta_k \sum_{k=1}^{K} SI_{k,t} + \beta_n \sum_{n=1}^{N} TA_{n,t} + u_t \rightarrow RES_t^{CMA};$$

$$RIRF_t^i = \alpha + \beta_1 RES_t^{MKT} + \beta_2 RES_t^{SMB} + \beta_3 RES_t^{HML} + \beta_4 RES_t^{RMW} + \beta_5 RES_t^{CMA} +$$
$$+ \beta_k \sum_{k=1}^{K} SI_{k,t} + \beta_n \sum_{n=1}^{N} TA_{n,t} + \varepsilon_t, \quad (3.20)$$

$$K = \{k_1, \dots, k_n | eigenvalue \geq 1\}; N = \{n_1, \dots, n_n | eigenvalue \geq 1\}.$$

Table 3.13 summarizes and compares results of R^2 values of the alternative and unified models for the US and Israel.

Table 3.13 Comparison between $\mathbf{R^2}$ values and its distribution of alternative and unified models

			R^2_{TASI}	R^2_{ALL}	$R^2_{ALL} - R^2_{TASI}$	$R^2_{5f} - R^2_{3f}$
1/2/2001-1/2/2017	US market - 50 securities	Min	0.43543	0.45318	0.00083	0.00061
		Max	0.67997	0.71854	0.09701	0.07039
		Average	0.54258	0.58216	0.03958	0.01746
	Israeli market - 14 securities	Min	0.30909	0.32613	-0.01368	N/A
		Max	0.70236	0.73271	0.04712	N/A
		Average	0.53257	0.56058	0.02801	N/A
			$R^2 > 0.5$	$R^2 > 0.55$	$R^2 > 0.6$	$R^2 > 0.7$
1/2/2001-1/2/2017	US market - 50 securities	Alternative	35	25	9	
		Unified	46	33	22	1
	Israeli market - 14 securities	Alternative	9	6	3	1
		Unified	11	8	6	2

Source: Own analysis

The alternative model demonstrates good performance. On the US market its explanatory power in absolute terms varies from 0.435 to almost 0.680 with the average of 0.543. However, the unified model still has the ability to improve alternative model, adding to its explanation between 0.010 and 0.100 with the average of 0.040. These results do not include 3 exceptions, where the unified model adds only 0.0058–0.0066 (KLAC and SYMC) and exception of VRTX, where the model contributes even less and close to 0.000. In this sense such contribution is even higher than those of five–factor model to three–factor model. On average this contribution is more than twice. Additionally, within the alternative model there are only 35 out of 50 cases (70%) where R^2 exceeds the 0.5, though the unified model suggests 42 cases (92%). On the Israeli market the alternative model demonstrates similar performance. In absolute terms its R^2 explains about 0.310 to 0.702 with the average of 0.533. In the Israeli case, the unified model still contributes between 0.017 to 0.047 adding almost 0.030 on average, which does not include the only case of ISRAMCO where the unified model loses 0.013 regarding the alternative model. These results are little low, but very comparable to those of the US market.

Table 3.14a demonstrates the distribution of alternative model coefficients by pattern and significance for the whole examined period on the US and Israeli markets. Table 3.14b in the Annex demonstrates average values of the coefficients for the alternative model. The column RES_t^{MKT} also reflects the results of RES_t^{TASI} on the Israeli market. Both parameters are connected to the market return.

Table 3.14a Results for alternative model for whole examined period in the US and Israel

$$RIRF_t^i = \alpha + \beta_1 TA1_t + \beta_2 TA2_t + \beta_3 TA3_t + (\beta_4 TA4_t) + \beta_5 SI1_t + (\beta_6 SI2_t) + \beta_7 (Mkt - RF)_t + \beta_8 SMB_t + \beta_9 HML_t + \beta_{10} RMW_t + \beta_{11} CMA_t + \varepsilon_t$$

		Technical indicators				Sentiment indicators		Fundamental indicators				
		$TA1_t$	$TA2_t$	$TA3_t$	$(TA4_t)$	$SI1_t$	$(SI2_t)$	RES_t^{MKT}	RES_t^{SMB}	RES_t^{HML}	RES_t^{RMW}	RES_t^{CMA}
Panel A: 1/2/2001-1/2/2017	Negative	50	50	50	16		24		23	44	33	25
	Positive				3	50	19	50	27	6	17	25
	NONE				31		7					
US market - 50 securities	1%	49	50	49	17	50	43	50	33	42	36	29
	5%				1				3	4	4	2
	10%			1					4	1	3	2
	10%>	1			1				10	3	7	17
Panel B: 1/2/2001-1/2/2006	Negative	14	14	13	14		9		N/A	N/A	N/A	N/A
	Positive			1		14	4	14	N/A	N/A	N/A	N/A
	NONE						1		N/A	N/A	N/A	N/A
Israeli market - 14 securities	1%	14	14	11	14	14	13	14	N/A	N/A	N/A	N/A
	5%								N/A	N/A	N/A	N/A
	10%								N/A	N/A	N/A	N/A
	10%>			3					N/A	N/A	N/A	N/A

Source: Own analysis

Regarding to the coefficients, there are only 2 cases in the US, where the alternative model has improvement relatively to the fundamental components, turning their coefficients to be significant at level of 1% or 5%, while within the unified model those coefficients are insignificant[33]. The coefficient pattern for sentiment and technical predictors is very stable though the 1st technical predictor turns to be negative, originally being positive (Tables 3.8a,b). The 2nd and 3rd predictors are negative as in original model and the 4th predictor may be negative or positive. In the case of MAR first technical predictor appears to be insignificant, originally being significant. For FISV its fourth technical predictor turns to insignificant. From the other side, the fourth predictor of BIIB turns to be significant at 5%. The sentiment predictors are much like in the original model. First predictor seems to be always significant and 2nd one can be positive or negative. In Israel 3 technical coefficients turn to be insignificant relatively to the original model. However, 3rd predictors of NICE and DELEK DRILL turns to be significant from the other side. Despite such fact, 3 insignificant coefficients are too much for such a small sample. The sentiment predictors demonstrate the same behavior as in the original model.

Examining the results of unified and alternative models, it is possible to figure out that unified model has some advantages over the alternative model though they are not so big. The unified model contributes to explanation of the alternative more even than five–factor model contributes to the explanation of the three–factor model. **For this reason, the unified model is still a better choice and should be preferred over the alternative one**.

3.4. Conclusions

The main purpose of this PhD thesis is to introduce and to test a Unified Capital Asset Pricing Model, which assumes that non–fundamental price is both technical and sentimental but not separate as the literature suggests. In turn, the model allows integration of the rational–based and non–rational–based approaches into one pricing mechanism, making the model first — universal, second — more appropriate in describing economic reality. Indeed, the results of the tests demonstrate that the unified model has a superiority over technical, sentiment or fundamental models and even over potential alternative model.

[33] Such statement refers only to cases where the change in the significance level is meaningful, i.e. at least 2 levels difference in Table 3.13. The change of only one level is not enough to determine either the change is meaningful.

There are 3 hypotheses in this study which have been verified empirically. The overall results and conclusions are as follows.

H1: Deviation components hypothesis

The results of the tests reveal that technical and behavioral models demonstrate very similar patterns in coefficients for all periods and markets. Their explanatory power separately is also similar. The integration of technical or behavioral components with relevant fundamental components obviously improves the coefficients' significance and explanatory power of both models for both examined markets during all periods. However, when the technical and behavioral components are integrated into the unified component, the performance improves impressively. The unified model improves every single model in almost every parameter for all periods and markets. The biggest contribution of the unified model is seen during the integration with the fundamental components. This indicates that non–fundamental component is much better explained in the terms of the unified model rather than in terms of the technical or behavioral models. Moreover, the non–fundamental component is explained better by the unified model than by the alternative model, where the technical and behavioral components inserted together but without integration into the unified component through the *PCA*. Despite that the unified model is a better choice than the alternative model, the last one has much better performance than the technical or behavioral models separately, adding to prove that the non–fundamental component should be explained in the terms of technical and behavioral components in contrast to what is accepted in the literature. From here, this hypothesis is successfully reached.

H2: Explanatory performance hypothesis

The explanatory power is reflected in the value of coefficient of explanation R^2. I consider the explanatory performance in 2 forms. The 1st form is the absolute values or contribution of the models to explanation expressed in the difference between absolute values of R^2 for two models. The 2nd form includes the number of companies that succeeded to exceed a given level of R^2 values, where $R^2 \geq 0.5$ is the threshold.

Regarding to the absolute values, the technical, behavioral and also fundamental models separately demonstrate lower performance than those of the unified model. All the models, except the unified one, exhibit relatively low average values of R^2 regarding to the daily returns. Once again, the values of R^2 for the technical and behavioral models are similar though the fundamental models demonstrate different values. On the US market the fundamental model demonstrates higher performance than those of the technical and sentiment models. It is also found that the contribution to explanation of the five–factor model comparing to the three–factor model is obviously not impressive. On the Israeli market the values of R^2 are absolutely comparable to those of the technical and sentiment models. From the other side, the values of R^2 for the unified model are much higher. From the beginning the model demonstrates permanently higher performance which is significantly improving with the integration of all components into one model. Such situation is true for both markets. Additionally, the alternative model has some impressive improvement in explanatory power comparing to the technical or sentiment, though the improvement of the unified model is still higher. The unified model contributes to explanation of the alternative model more than twice on average than the contribution of five–factor model to those of the three–factor one.

Regarding to the number of companies that succeeded to exceed a given level of R^2 values, it mostly refers to the integrated models. The results demonstrate that on both markets the technical and sentiment models are able to exceed 0.4 and rarely to exceed 0.5, which is not enough. The highest results for all models and markets recorded in Panel C. In contrast, the unified model on both markets easily exceeds 0.5 and in the vast majority of the cases it exceeds 0.55. In about a half or more of all cases the R^2 values of the unified model exceed even 0.6, which is much more impressive comparing to the previous models. In comparison to the alternative model, such parameter also stands in a favor of the unified model, though the alternative model demonstrates better results than those of technical or sentiment. Unfortunately, examining the dynamics through the subperiods we may see that this parameter declines with time, suggesting that in larger samples there will be less companies with values of R^2 exceeding the threshold. However, it is unclear whether the reason for such decline is a period length itself or economic shocks included in this period (reminding the sub–prime crisis is included in the examined period). All these indicate that this hypothesis is successfully reached.

H3: Significance hypothesis

This hypothesis claims that all parts of the unified model should be significant. The technical and behavioral components, derived from the *PCA* are significant at the highest level with several exceptions. The integration of fundamental components raises the level of significance of all models, including the unified one, during all periods for both markets. Although, the unified model has proportionally more insignificant coefficients comparing to other models. This mostly happens as a result of inheritance of the significance from the technical or sentiment models, though the unified model is composed of technical and behavioral components. Despite this fact, the insignificant coefficients have extremely low values and can be dropped without to harm explanatory power. However, the goal is to create a universal model on a basis of a universal rule, hence the insignificant coefficients were not dropped. From here, this hypothesis is partly reached, however can be improved in the future.

Final remarks

The idea of capital asset pricing is more than one hundred years. Determining capital asset prices' behavior is important to individuals in reducing the uncertainty, insuring their savings. In order to answer such need researchers attempt to introduce a model or theory that has the ability to describe the financial reality the best way. However, this task is complicated. During all this time a lot of scientists demonstrate brilliant thinking and approaches though until today there is no unified financial theory, which is paradoxical. Probably a reason for such complicity comes out from a difficulty to describe a human behavior as a whole. Hence, the problem is reduced to searching for a sufficient proxy to model such behavior. This in general led to the foundation of two main competitive schools. First school is classical with normative approach, based on economic parameters. In contrast, second school developed behavioral approach, involving non–economic parameters.

The competition between the main schools is so strong that there is no cooperation between them. The normativists and behaviorists disagree in every principal point of view. Normative theory is solid and exists much longer than the behavioral, though behavioral theory is younger and more dynamic. There are so many differences between the approaches that it looks like each theory is applied to different individuals. In addition, the models proposed by the normativists and behaviorists may work only in some samples, in some countries or during some specific periods. The methodologies, implemented in the studies, lead to huge deviation and variation in the results of those studies, adding to the disagreement even more. Hence, the need to develop more universal financial theoretical frame is palpable. Meanwhile, both schools have accumulated enough scientific experience to make a step to a creation of such universal financial platform. However, even an attempt has not been yet done.

The Unified Capital Asset Pricing Model, proposed and tested in this thesis, is called up to fill the gap between the normativists and the behaviorists. It assumes that the integration of best achievements from both schools should lead to one solid financial theory that is also unified and universal and has a better performance than normative or behavioral approaches separately. I believe that such integration indeed can be done. The task is still complicated, but it does not mean impossible to improve the financial theory as a whole.

The Unified Capital Asset Pricing Model answers the hypothesizes and goals proposed in this study. From the results of the tests it is possible to conclude that the unified model indeed constitutes real improvement, though in some lower degree than expected. It has a better explanatory power than every single model separately or potential alternative model on the background of stable coefficients. It demonstrates that the *PCA* methodology can be useful with 2 conditions: the variables used are proved to be relevant; the influence of every single variable is not important but their overall influence. Also, there is an acute need to improve the sentiment variables, searching for more relevant and qualitive variables to enlarge their number.

Comparing the results for the US and the Israeli markets, it is possible to figure out that general picture is very similar for both markets. All the models exhibit similar performance regarding the coefficient values and pattern, significance or explanatory power. Those results are consistent from the beginning during all subperiods. All this suggests that such models work on international level and can be suitable with other markets, making the unified model even more universal. This point was out of goals of the study though deserves to be investigated deeper.

Since the unified model can improve explanatory power of the daily returns, further investigation of the model is useful and grateful. In the future research, the results of the unified model should be validated on international markets. The unified model should be tested on monthly average returns and much larger samples, which are much more suitable with the fundamental models, including more relevant sentiment variables involved. If some researcher does that and he gets the similar results as in this PhD thesis then the Unified Capital Asset Pricing Model can be another step in the long and fascinating history of the world of financial theory to a better understanding of the fundamental and non–fundamental factors creating a pattern of prices fluctuations.

Reference

Abbas, N., Khan, J., Aziz, R., & Sumrani, Z. (2015). A study to check the applicability of Fama and French, three–factor model on KSE 100–index from 2004–2014. *International Journal of Financial Research, 6*(1), 90–100.

Agrawal, A., & Tandon, K. (1994). Anomalies or illusions? Evidence from stock markets in eighteen countries. *Journal of International Money and Finance, 13*(1), 83–106.

Ajili, S. (2002). *The capital asset pricing model and the three factor model of Fama and French revisited in the case of France* (CEREG University of Paris Working Paper, No. 2002–10).

Ajili, S. (2003). *Explaining the cross–section returns in France: Characteristics or covariances?* CEREG University of Paris Working Paper. DOI: 10.2139/ssrn.391565

Akemann, C.A., & Keller, W.E. (1977). Relative Strength does persist! *The Journal of Portfolio Management, 4*(1), 38–45.

Akerlof, G.A. (1970). The market for lemons: Quality uncertainty and the market mechanism. *Quarterly Journal of Economics, The MIT Press,* 84(3), 488–500.

Alexander, S.S. (1961). Price movements in speculative markets: Trends or random walks. *Industrial Management Review, 2*(2), 7–26.

Alexander, S.S. (1964). Price movements in speculative markets: Trends or random walks No. 2. *Industrial Management Review, 5*(2), 25–46.

Allais, M.F.C. (1953). Le comportement de l'homme rationnel devant le risque: Critique des postulats et axiomes de l'ecole americaine. *Econometrica, 21*(4), 503–546.

Allen, D.E., & Cleary. F. (1998). Determinants of the cross–section of stock returns in the Malaysian stock market. *International Review of Financial Analysis, 7*(3), 253–275.

Allen, R.G.D., & Hicks, J.R. (1934). A reconsideration of the theory of value, II. *Economica, 1*(2), 196–219.

Alwathainani, A.M. (2012) Market reaction to an earnings shock: A test of the conservatism effect, *Journal of Behavioral Finance & Economics, 2*(2), 14–37.

Amihud, Y. (2002). Illiquidity and stock returns: Cross–section and time–series effects, *Journal of Financial Markets, 5*(1), 31–56.

Amihud, Y., & Mendelson, H. (1986). Asset pricing and the bid–ask spread. *Journal of Financial Economics, 17*(2), 223–249.

Amihud, Y., & Mendelson, H. (1989). The effects of beta, bid–ask spread, residual risk, and size on stock returns, *Journal of Finance, 44*(2), 479–486.

Anderson, T.W. (1984). *An introduction to multivariate statistical analysis*. New York: John Wiley & Sons.

Antweiler, W., & Frank, M. (2004). Is all that talk just noise? The information content of internet stock message boards. *Journal of Finance, 59*(3), 1259–1295.

Arnott, R.D. (1979). Relative strength revisited. *Journal of Portfolio Management, 5*(3), 19–23.

Arrow, K.J., & Debreu, G. (1954). Existence of an equilibrium for a competitive economy, *Econometrica, 22*(3), 265–290.

Asness, C.S. (1994). *Variables that Explain Stock Returns* (Doctoral Dissertation, University of Chicago).

Ataullah, A. (2001). Macroeconomic variables as common pervasive risk factors and the empirical content of the arbitrage pricing theory. *Journal of Empirical finance, 5*(3), 221–240.

Bajgrowicz, P., & Scaillet, O. (2012). Technical trading revisited: false discoveries, persistence tests, and transaction costs. *Journal of Financial Economics, 106*(3), 473–491.

Bahl, B. (2006). *Testing the Fama and French three–factor model and its variations for the Indian stock returns* (SSRN Working Pares Series). DOI: 10.2139/ssrn.950899.

Baker, M.P., & Wurgler, J.A. (2006). Investor sentiment and the cross–section of stock returns. *Journal of Finance, 61*(4), 1645–1680.

Baker, M.P., & Wurgler, J.A. (2007). Investor sentiment and the stock market. *Journal of Economic Perspectives, 21*(2), 129–151.

Baker, M.P., Wurgler, J.A., & Yuan, Y. (2012). Global, local, and contagious investor sentiment. *Journal of Financial Economics, 104*(2), 272–287.

Ball, R., & Brown, P. (1968). An empirical evaluation of accounting income numbers. *Journal of Accounting Research, 6*(2), 159–178.

Banz, R. (1981). The relationship between return and market value of common stock. *Journal of Financial Economics, 9*(1), 3−18.

Barberis, N., Shleifer, A., & Vishny, R. (1998). A model of investor sentiment. *Journal of Financial Economics, 49*(3), 307–343.

Bartholdy, J., & Peare, P. (2005). Estimation of expected return: CAPM vs. Fama and French*, International Review of Financial Analysis, 14*(4), 407–427. DOI: 10.1016/j.irfa.2004.10.009 · .

Basu, S. (1977). Investment performance of common stocks in relation to their price–earnings ratio: A test of the efficient market hypothesis. *Journal of Finance, 32*(3), 663–682.

Basu, S. (1983). The relationship between earnings' yield, market value and return for NYSE common stocks: Further evidence. *Journal of Financial Economics, 12*(1), 129–156.

Bawa, V., & Lindenberg, E. (1977). Capital market equilibrium in a mean–lower partial moment framework. *Journal of Financial Economics, 5*(2), 189–200.

Bell, D. (1982) Regret in decision making under uncertainty. *Operations Research, 30*(5), 961−981.

Bem, D.J. (1965). An experimental analysis of self–persuasion. *Journal of Experimental Social Psychology, 1*(3), 199–218.

Bentham, J. (1789). *An introduction to the principles of morals and legislation*. London, UK: Jonathan Bennett.

Berk, J.B. (2000). Sorting out sorts. *Journal of Finance, 55*(1), 407–427.

Bernard, V. (1992). Stock Price Reactions to Earnings Announcements. In R., Thaler (Ed.). *Advances in Behavioral Finance* (pp. 303–340). New York: Russell Sage Foundation.

Bernard, V., & Thomas, J. (1990). Evidence that stock prices do not fully reflect the implications of current earnings for future earnings. *Journal of Accounting and Economics, 13*(4), 305–341

Bernoulli, D. (1738). Exposition of a new theory on the measurement of risk, translated by Dr. Louise Sommer in 1954. *Econometrica, 22*(1), 22–36.

Bessembinder, H., & Chan, K. (1995). The profitability of technical trading rules in the Asian stock markets. *Pacific–Basin Finance Journal, 3*(2), 257–284.

Bessembinder H., & Chan, K. (1998). Market efficiency and the returns to technical analysis. *Financial Management, 27*(2), 5–17.

Bhandari, L.C. (1988). Debt/equity ratio and expected common stock returns: Empirical evidence. *Journal of Finance, 43*(2), 507–28.

Bilinski, P., & Lyssimachou, D. (2004). *Validating the Fama & French three factor model; the case of the Stockholm stock exchange 1982 – 2002* (Master's Thesis, Goteborg University School of Economics and Commercial Law, Industrial and Financial Economics.

Black, F. (1972). Capital market equilibrium with restricted borrowing. *Journal of Business, 45*(3), 444–455.

Black, F. (1986). Noise. *Journal of Financial Economics, 41*(3), 528–543.

Black, F. (1993). Beta and return*. Journal of Portfolio Management, 20*(1), 8–18.

Black, F., Jensen, M.C., & Scholes M. (1972). The capital asset pricing model: Some empirical tests In M. Jensen (Ed.). *Studies in the Theory of Capital Markets* (pp. 79–121). New York, NY: Praeger.

Blitz, D., & Vidojevic, M. (2017). The profitability of low–volatility. *Journal of Empirical Finance, Elsevier, 43*(C), 33–42.

Bloomfield, R.J., & Hales, J. (2002). Predicting the next step of a random walk: Experimental evidence of regime–shifting beliefs. *Journal of Financial Economics, 65*(3), 397–414.

Bloomfield, R.J., Taylor, W.B., & Zhou, F. (2009). Momentum, reversal, and uninformed traders in laboratory markets. *Journal of Finance, 64*(6), 2535–2558.

Blume, M. (1970). Portfolio theory: A step towards its practical application. *Journal of Business, 43*(2), 152–174.

Blume, M., & Friend, I. (1973). A new look at the capital asset pricing model. *Journal of Finance, 28*(1), 19–33.

Bohan, J. (1981). Relative strength: further positive evidence. *The Journal of Portfolio Management, 8*(1), 36–39.

Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. *Review of Economics and Statistics, 69*(3), 542–547.

Bollerslev, T., Engle, R.F., & Wooldridge, J.M. (1988). A capital asset pricing model with time–varying covariances. *Journal of Political Economy, 96*(1), 116–131.

Bornholt, G.N. (2007). Extending the capital asset pricing model: The reward beta approach. *Journal of Accounting and Finance,47*(1), 69–83.

Bower, D., Bower, R., & Logue, D. (1984). Arbitrage Pricing and Utility Stock Returns. *Journal of Finance, 39*(4), 1041–1054.

Breeden, D.T. (1979). An inter–temporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics, 7*(3), 265–296.

Breeden, D.T., Gibbons, M.R., & Litzenberger, R.H. (1989). empirical tests of the consumption–oriented CAPM. *Journal of Finance, 44*(2), 231–262.

Breeden, D.T., Litzenberger, R.H., & Jia, T. (2015). Consumption–based asset pricing, part 1: Classic theory and tests, measurement issues, and limited participation. *Annual Review of Financial Economics, 7*(1), 35–83.

Brennan, M.J., & Subrahmanyam, A. (1996). Market microstructure and asset pricing: On the compensation for illiquidity in stock returns. *Journal of Financial Economics, 41*(3), 441–464.
Brock W., Lakonishok J., & LeBaron B. (1992). Simple technical trading rules and the stochastic properties of stock returns. *Journal of Finance, 47*(5), 1731–1764.
Brown, G.W. (1999). Volatility, sentiment, and noise traders. *Financial Analysts Journal, 55*(2), 82–90.
Brown, G.W., & Cliff, M.T. (2004). Investor sentiment and the near–term stock market, *Journal of Empirical Finance, 11*(1), 1–27.
Brown, G.W., & Cliff, M.T. (2005). Investor sentiment and asset valuation. *Journal of Business, 78*(2), 405–440.
Brown, P., Kleidon, A.W., & Marsh, T.A. (1983). New evidence on the nature of size related anomalies in stock prices. *Journal of Financial Economics, 12*(1), 33–56.
Brown, S. (1989). The number of factors in security returns. *Journal of Finance, 44*(5), 1247–1262.
Brown, S., Weinstein, M. (1983). A new approach to testing asset pricing models. *Journal of Finance, 38*(3), 711–743.
Brush, J.S. (1986). Eight relative strength models compared. *Journal of Portfolio Management,* 13(1), 21–28.
Brush, J.S., & Boles, K.E. (1983). The predictive power in relative strength & CAPM. *The Journal of Portfolio Management, 9*(4), 20–23.
Burmeister, E., McElroy, M. (1988). Joint estimation of factor sensitivities and risk premia for the arbitrage pricing theory. *Journal of Finance, 43*(3), 721–733.
Burmeister, E., & Wall, K.D. (1986). The arbitrage pricing theory and macroeconomic factor measures. *Financial Review, 21*(1), 1–20.
Caginalp, G., & Laurent, H. (1998). The predictive power of price patterns. *Applied Mathematical Finance, 5*(3–4), 181–205.
Campbell, J.Y. (1993). Intertemporal asset pricing without consumption data. *American Economic Review, 83*(3), 487–512.
Campbell, J.Y. (1996). Understanding risk and return. *Journal of Political Economy, 104*(2), 298–345.
Campbell, J.Y., Hilscher, J.D., & Szilagyi, J. (2008). In Search of Distress Risk. *Journal of Finance, American Finance Association, 63*(6), 2899–2939.
Carhart, M.M. (1997). On persistence in mutual fund performance. *Journal of Finance, 52*(1), 57–82.
Chamberlain, G., & Rothschild, M. (1983). Arbitrage, factor structure, and mean variance analysis on large asset markets. *Econometrica, 51*(5), 1281–1304.
Chan, L.K.C., Hamao, Y., & Lakonishok, J. (1991). Fundamentals and stock returns in Japan. *Journal of Finance, 46*(5), 1739–1764.
Chan, L.K.C., Jegadeesh, N., & Lakonishok, J. (1996). Momentum strategies. *Journal of Finance, 51*(5), 1681–1713.
Chan, L.K.C., & Lakonishok, J. (2004). Value and growth investing: review and update. *Financial Analysts Journal, 60*(1), 71–86.
Chan, W., Frankel, R., & Kothari, S.P. (2004). Testing behavioral finance theories using trends and sequences in financial performance. *Journal of Accounting and Economics, 38*(1), 3–50.
Chang, C., Faff, R.W., & Hwang, C.–Y. (2009). *Sentiment contagion, corporate governance, information and legal environments* (SSRN Working paper). http://dx.doi.org/10.2139/ssrn.1359351.
Chang, P.H.K., & Osler, C.L. (1999). Methodical madness: Technical analysis and the irrationality of exchange–rate forecasts. *Economic Journal, 109*(458), 636–661.
Charoenrook, A. (2005). *Does sentiment matter?* (Vanderbilt University, Working Paper).
Charitou, A., & Constantinidis, E. (2004). *Size and book–to–market factors in earnings and stock returns: Empirical evidence for Japan* (University of Cyprus, Working Paper).
Chen, N.F. (1983). Some empirical tests of the theory of arbitrage pricing. *Journal of Finance, 38*(5), 1393–1414.
Chen, H., De, P., Hu, Y., & Hwang, B.–H. (2012). *Customers as advisors: The role of social media in financial markets* (SSRN Working Paper).
Chen, H., De, P., Hu, Y., & Hwang, B. –H. (2014). Wisdom of crowds: The value of stock opinions transmitted through social media. *Review of Financial Studies, 27*(5), 1367–1403.
Chen, H., Chong, T.T.–L., & Duan, X. (2010). A principal–component approach to measuring investor sentiment. *Quantitative Finance, 10*(4), 339–347.
Chen, N.F., Kan, R., & Miller, M.H. (1993). Are the discounts on closed–end funds a sentiment index? *Journal of Finance, 48*(2), 795–800.
Chen, N.F., Roll, R., Ross, S.A. (1986). Economic forces and the stock market. *Journal of Business, 59*(3), 383–403.
Cheng, A.C.S. (1993). *International arbitrage pricing theory: Empirical evidence from the United Kingdom and the United States* (Doctoral Dissertation, London School of Economics and Political Science (UK)).
Cheng, A.C.S. (1995). The UK Stock market and economic factors: A new approach. *Journal of Business Finance*

and Accounting, 22(1), 129–142.
Chew, S.H. (1983). A generalization of the quasilinear mean with applications to the measurement of income inequality and decision theory resolving the Allais paradox. *Econometrica, 51*(3), 1065−1092.
Chew, S.H. (1989). Axiomatic utility theories with the betweenness property. *Annals of Operations Research, 19*(1−4), 273−98.
Chiah, M., Chai, D., & Zhong, A. (2016). A better model? An empirical investigation of the Fama–French five–factor model in Australia. *International Review of Finance, 16*(4), 595–638.
Cho, D.C. (1984). On testing the arbitrage pricing theory: Inter–battery factor analysis*. Journal of Finance, 39*(5), 1485–1500.
Cho, D.C., Elton, E.J., & Gruber, M.J. (1984). On the robustness of the Roll and Ross arbitrage pricing theory. *Journal of Financial and Quantitative Analysis, 19*(1), 1–10.
Cho, D.C., Taylor, W.M. (1987). The seasonal stability of the factor structure of stock returns. *Journal of Finance, 42*(5), 1195–1211.
Chopra, N., Lee, C.M.C., Shleifer, A., & Thaler, R.H. (1993). Yes, discounts on closed–end funds are a sentiment index. *Journal of Finance, 48*(2), 801–808.
Chordia, T., Subrahmanyam, A., & Anshuman, V.R. (2001). Trading activity and expected stock returns. *Journal of Financial Economics, 59*(1), 3–32.
Chou, R.Y. (1988). Volatility persistence and stock valuations: Some empirical evidence using garch. *Journal of Applied Econometrics, 3*(4), 137–167.
Chou, P., Li, W., & Zhou, G. (2004). Portfolio optimization under asset pricing anomalies. *Japan and the World Economy, 18*(2), 121–142.
Christ, K.P., & Bremmer, D.S. (2003). The Relationship Between Consumer Sentiment and Stock Prices. Paper presented at the *78th Annual Conference Proceedings of the Western Economics Association International*.
Chui, A.C.W., Titman, S., & Wei, K.C.J. (2003). Intra–industry momentum: The case of REITs. *Journal of Financial Markets, 6*(3), 363–387.
Chui, A.C.W., & Wei, K.C.J. (1998). Book–to–market, firm size, and the turn–of–the–year effect: Evidence from pacific–basin emerging markets. *Pacific–Basin Finance Journal, 6*(3), 275–293.
Clarice, M., & William, E.Jr. (2015). Pricing assets with Fama and French 5–factor model: A Brazilian market novelty. *Research Gate*, 1–13. Retrieved from: https://www.researchgate.net/publication/277020668.
Clark, P.K. (1973). A subordinated stochastic process model with finite variance for speculative prices. *Econometrica, 41*(1), 135–155.
Clarke R.G., & Statman, M. (1998). Bullish or bearish? *Financial Analysts Journal, 54*(3), 63–72.
Cochrane, J.H. (2005). *Asset Pricing*. Princeton, US and Oxford, UK: Princeton University Press.
Connor, G., & Korajczyk, R. (1986). performance measurement with the arbitrage pricing theory: A new framework for analysis. *Journal of Financial Economics, 15*(3), 373–394.
Connor, G., & Korajczyk, R. (1988). Risk and return in an equilibrium APT: Application of a new test methodology. *Journal of Financial Economics, 21*(2), 255–289.
Connor, G., & Sehgal, S. (2001). *Tests of the Fama and French Model in India* (Discussion Paper, No. 379). Financial Markets Group, Retrieved from London School of Economics and Political Science website: http://eprints.lse.ac.uk/id/eprint/25057.
Conrad, J., & Kaul, G. (1998). An anatomy of trading strategies. *Review of Financial Studies, 11*(3), 489–519.
Constantinides, G.M. (1990). Habit formation: A resolution of the equity premium puzzle. *Journal of Political Economy, 98*(3), 519–543.
Cootner, P.H. (1962). Stock prices: Random vs. systematic changes. *Industrial Management Review, 3*(2), 24–45.
Cornell, W.B., & Dietrich, J.K. (1978). The efficiency of the market for foreign exchange under floating exchange rates. *The Review of Economics and Statistics, 60*(1), 111–120.
Coutts, J. A., & Cheung, K. (2000). Trading rules and stock returns: Some preliminary short run evidence from the Hang Seng 1985–1997. *Applied Financial Economics, 10*(6), 579–586.
Cowles, A. (1938). *Common Stock Indexes 1871–1937. The American Economic Review, 29*(1), 205–207.
Cremers, M., Petajisto, A., & Zitzewitz, E. (2013). Should benchmark indices have alpha? Revisiting performance evaluation. *Critical Finance Review, 2*(1), 1–48.
Cutler, D., Poterba, J., & Summers, L. (1991). Speculative dynamics. *Review of Economic Studies, 58*(3), 529–546.
Dale, C., & Workman, R. (1980). The arc sine law and the treasury bill futures market. *Financial Analysts Journal,* 36(6), 71–74.
Daniel, K. (2004). Discussion of: ‘‘Testing behavioral finance theories using trends and sequences in financial performance,’’ (by Wesley Chan, Richard Frankel, and S.P. Kothari). *Journal of Accounting and*

Economics, 38(1), 51–64.
Daniel, K., Hirshleifer, D., & Subrahmanyam, A. (1998). Investor Psychology and Security Market Under– and Overreactions. *Journal of Finance, 53*(6), 1839–1885.
Daniel, K., & Titman, S. (1997). Evidence on the characteristics of cross sectional variation in stock returns, *Journal of Finance, 52*(1), 1–33.
Daniel, K., & Titman, S. (1999). Market efficiency in an irrational world. *Financial Analyst Journal, 55*(6), 28–40.
Daniel, K., Titman, S., & Wei, K.C.J. (2001). Explaining the cross–section of stock returns in Japan: factors or characteristics? *Journal of Finance, 56*(2), 743–766.
Das, S.R., & Chen, M.Y. (2007). Yahoo! for Amazon: Sentiment extraction from small talk on the web, *Management Science, 53*(9), 1375–1388.
Datar, V., Naik, N., & Radcliffe, R. (1998). Liquidity and asset returns: An alternative test. *Journal of Financial Markets, 1*(2), 203−219.
Davis, J.L., Fama, E.F., & French, K. (2000). Characteristics, covariances and average returns. *Journal of Finance, 55*(1), 389−406.
Dawson, E.R., & Steeley, J., (2003). On the existence of visual technical patterns in the UK stock market. *Journal of Business Finance and Accounting, 30*(1–2), 263–293.
DeBondt, W., & Thaler, R. (1985). Does the stock market overreact? *Journal of Finance, 40*(3), 793–808.
DeBondt, W., & Thaler, R. (1987). Further evidence of investor overreaction and stock market seasonality, *Journal of Finance, 42*(3), 557–581.
De Brouwer, P. (2009). Maslowian portfolio theory: An alternative formulation of the behavioral portfolio theory. *Journal of Asset Management, 9*(6), 359–365.
Dechow, P.M., Hutton, A.P., & Sloan, R.G. (1999). An empirical assessment of the residual income valuation model1. *Journal of Accounting and Economics, 26*(1–3), 1–34.
Dekel, E. (1986). An axiomatic characterization of preferences under uncertainty: Weakening the independence axiom. *Journal of Economic Theory, 40*(2), 304−18.
De la Vega, J. (1688). *Confusion of Confusions*. Santa Cruz, US: Selected and Translated by Professor Herman Kellenbenz, University of California.
De Long, B.J., Shleifer, A., Summers, L.H., & Waldmann, R.J. (1991). The survival of noise traders in financial markets. *Journal of Business, 64*(1), 1–19.
Dewally, M. (2003). Internet investment advice: Investing with a rock of salt, *Financial Analysts Journal, 59*(4), 65–77.
Dhrymes, P.J. (1984). The empirical relevance of arbitrage pricing models. *Journal of Portfolio Management, 10*(4), 35–44.
Dhrymes, P.J., Friend, I., & Gultekin, N.B. (1984). A critical reexamination of the empirical evidence on the arbitrage pricing theory. *Journal of Finance, 39*(2), 323–346.
Dhrymes, P.J., Friend, I., Gultekin, N.B., & Gultekin, M.N. (1985a). An empirical examination of the implications of APT. *Journal of Banking and Finance, 9*(1), 73–99.
Dhrymes, P.J., Friend, I., Gultekin, N.B., & Gultekin, M.N. (1985b). New tests of the APT and their implications. *Journal of Finance, 40*(3), 659–674.
Dimson, E., & Marsh, P. (1999). Murphy's law and market anomalies. *Journal of Portfolio Management, 25*(2), 53–69.
Dimson, E., Marsh, P., & Staunton, M. (2014). *Credit Suisse global investment returns yearbook 2014*. Zurich, Swiss: Credit Suisse Research Institute.
Donchian, R.D. (1960). Commodities: High finance in copper. *Financial Analysts Journal, 16*(6), 133–142.
Douglas, G.W. (1968). *Risk in the Equity Markets: An Empirical Appraisal of Market Efficiency*. Ann Arbor, Michigan: University Microfilms.
Doukas, J.A., & McKnight, P.J. (2005). European momentum strategies, information diffusion, and investor conservatism. *European Financial Management, 11*(3), 313–38.
Dybvig, P., Ross, S.A. (1985). Yes, the APT is testable. *Journal of Finance, 40*(4), 1173–1188.
Edelen, R.M., Marcus, A.J., & Tehranian, H. (2010). Relative sentiment and stock returns. *Financial Analysts Journal, 66*(4), 20–32.
Edwards, R.D., & Magee, J. (Eds.). (1948). *Technical analysis of stock trends* (9th ed.). San Francisco, CA: CRC Press.
Edwards, W. (1968). Conservatism in human information processing. In B., Kleinmutz (Ed.). *Formal Representation of Human Judgment* (pp. 17–52). New York, NY: John Wiley & Sons.
Efron, B. (1979). Bootstrap methods: Another look at the jackknife. *The Annals of Statistics, 7*(1), 1–26.
Elfakhani, S., Lockwood, L.J., & Zaher, T.S. (1998). Small firm and value effects in the Canadian stock market.

Journal of Financial Research, 21(3), 277–291.
Eleswarapu, V.R., & Reinganum, M.R. (1993). The seasonal behavior of liquidity premium in asset pricing. *Journal of Financial Economics, 34*(3), 373–386.
Elton, E.J., Gruber, M.J., & Busse, J.A. (1998). Do investor care about sentiment? *Journal of Business, 71*(4), 477–500.
Estrada, J. (2002). Systematic risk in emerging markets: The D–CAPM. *Emerging Market Review, 3*(4), 365–379.
Estrada, J. (2007). Mean–semivariance behavior: Downside risk and capital asset pricing. *International Review of Economics Finance, 16*(2), 169–185.
Fabozzi, F.J., Cakici, N., & Tan, S. (2013). Size, Value, and Momentum in Emerging Market Stock Returns *Emerging Markets Review, 16*(C), 46–65.
Faff, R.W. (1992). A multivariate test of an equilibrium APT with time varying risk and risk premia in the Australian equity market. *Australian Journal of Management, 17*(2), 233–258.
Faff, R.W. (2001). A multivariate test of a dual–beta CAPM: Australian evidence. *Financial Review, 36*(4), 157–174.
Fama, E.F. (1965). The behavior of stock–market prices. *Journal of Business, 38*(1), 34–105.
Fama, E.F. (1970). Efficient capital markets: A review of theory and empirical work. *Journal of Finance, 25*(2), 383–417.
Fama, E.F. (1991). Efficient capital markets: II. *Journal of Finance, 46*(5), 1575–1617.
Fama, E.F. (1998). Market efficiency, long–term returns, and behavioral finance. *Journal of Financial Economics, 49*(3), 283–306.
Fama, E.F., & Blume, M.E. (1966). Filter rules and stock market trading. *Journal of Business, 39*(1), 226–241.
Fama, E.F., & French, K. (1988). Dividend yields and expected stock returns. *Journal of Financial Economics, 22*(1), 3−25.
Fama, E.F., & French, K.R. (1992). The cross–section of expected stock returns. *Journal of Finance, 47*(2), 427–465.
Fama, E.F., & French, K.R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics, 33*(1), 3–56.
Fama, E.F., & French, K.R. (1996). Multifactor explanations of asset pricing anomalies. *Journal of Finance, 51*(1), 55–84.
Fama, E.F., & French, K.R. (1998). Value versus growth: The international evidence. *Journal of Finance, 53*(6), 1975−1999.
Fama, E.F., & French, K.R. (2004). The capital asset pricing model: Theory and evidence. *Journal of Economic Perspectives, 18*(3), 25–46.
Fama, E.F., & French, K.R. (2008). Dissecting anomalies. *Journal of Finance, 63*(4), 1653–1678.
Fama, E.F., & French, K.R. (2015). A Five–Factor Asset Pricing Model. *Journal of Financial Economics*, *116*(1), 1–22.
Fama, E.F., & French, K.R. (2016a). Dissecting anomalies with a five–factor mode. *Review of Financial Studies, 29*(1), 69–103.
Fama, E.F., & French, K.R. (2017). International tests of a five–factor asset pricing model. *Journal of Financial Economics, 123*(3), 441–670.
Fama, E.F., & Macbeth, J. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy, 81*(3), 607–636.
Fama, E.F., & Macbeth, J. (1974). Tests of the multiperiod two parameter model. *Journal of Financial Economics, 1*(5), 43–66.
Febrian, E., & Herwany, A. (2010). CAPM and APT validation test before, during, and after financial crisis in emerging market, evidence from Indonesia. *The International Journal of Business and Finance Research, 10*(1), 85–97.
Ferguson, M.F., & Shockley, R.L. (2003). Equilibrium "anomalies". *Journal of Finance, 58*(6), 2549–2580.
Fernández–Rodríguez, F., González–Martel, C., & Sosvilla–Rivero, S. (2000). On the profitability of technical trading rules based on artificial neural networks: Evidence from the Madrid stock market. *Economic Letters, 69*(1), 89–94.
Fernández–Rodríguez, F., Sosvilla–Rivero, S., & Andrada–Félix, J. (2003). Technical analysis in foreign exchange markets: Evidence from the EMS. *Applied Financial Economics, 13*(2), 113–122.
Fisher, I. (1906). *The nature of capital and income*. New York: Cosimo.
Fisher, K.L., & Statman, M. (2000). Investor Sentiment and Stock Returns. *Financial Analysts Journal, 56*(2), 16–23.
Fisher, K.L., & Statman, M. (2003). Consumer Confidence and Stock Returns. *Journal of Portfolio Management,*

30(1), 115–127.
Fisher, K.L., & Statman, M. (2004). Sentiment, value, and market timing. *Journal of Investing, 13*(3), 10–21
Fogler, H., John, K., & Tipton, J. (1981). Three factors, interest rates differentials and stock groups. *Journal of Finance, 36*(5), 323–35.
Frankel, R., & Lee, C. (1998). Accounting valuation, market expectation, and cross–sectional stock returns. *Journal of Accounting and Economics, 25*(3), 283–319.
French, K.R. (1980). Stock Returns and the Weekend Effect. *Journal of Financial Economics, 8*(1), 55−69.
Friedman, M., & Savage, L.J. (1948). Utility analysis of choices involving risk. *Journal of Political Economy, 56*(4), 279–304.
Friedman, M. (1953). *The Case for Flexible Exchange Rates*. In Essays in Positive Economics, Chicago: University of Chicago Press.
Friend, I., & Blume, M. (1970). Measurement of portfolio performance under uncertainty. *American Economic Review, 60*(4), 607–636.
Fu, H.–P. (2016). Testing explanatory power of two behavioral models for price continuation. *International Journal of Financial Research, 7*(2), 93–97.
Garvett, I. & Priestly, R. (1997). Do assumption about factor structure matter in empirical test of APT? *Journal of Business finance and Accounting, 24*(2), 249–260.
Gaunt, C. (2004). Size and book to market effects and the Fama French three factor asset pricing model: Evidence from the Australian stock market. *Accounting and Finance, 44*(1), 27–44.
Gehr, A. (1978). Some tests of the arbitrage pricing theory. *Journal of the Midwest Finance Association, 7*(4), 91–95.
Gençay, R. (1998a). Optimization of technical trading strategies and the profitability in security markets. *Economic Letters, 59*(2), 249–254.
Gençay, R. (1998b). The predictability of security returns with simple technical trading rules. *Journal of Empirical Finance, 5*(4), 347–359.
Gençay, R. (1999). Linear, nonlinear and essential foreign exchange rate prediction with simple technical trading rules. *Journal of International Economics, 47*(1), 91–107.
Gençay, R., & Stengos, T. (1998). Moving average rules, volume and the predictability of security returns with feedforward networks. *Journal of Forecasting, 17*(5–6), 401–414.
Gibbons, M.R. (1982). Multivariate tests of financial models: A new approach. *Journal of Financial Economics, 10*(1), 3–27.
Gibbons, M.R., Ross, S.A., & Shanken, J. (1989). A test of the efficiency of a given portfolio. *Econometrica, 57*(5), 1121–152.
Gilovich, T., Vallone, R., & Tversky, A. (1985). The hot hand in basketball: on the misperception of random sequences. *Cognitive Psychology, 17*(1), 295–314.
Graham, B., & Dodd, D.L. (1934). *Security analysis* (1st ed.). New York, NY: The McGraw–Hill Companies.
Granville, J. (1963). *Granville's new key to stock market profits.* New York, NY: Prentice–Hall.
Griffin, D., & Tversky, A. (1992). The weighing of evidence and the determinants of confidence. *Cognitive Psychology, 24*(3), 411–435.
Groenewold, N., & Fraser, P. (1997). Share prices and macroeconomic factors. *Journal of Business Finance and Accounting, 24*(9), 1367–1383.
Groot, W., Pang, J., & Swinkels, L. (2012). The cross–section of stock returns in frontier emerging markets. *Journal of Empirical Finance, 19*(5), 796–818.
Guillaume, D.M. (2000). *Intradaily exchange rate movements*. Boston, MA: Kluwer Academic Publishers.
Gultekin, M.N., & Gultekin, N.B. (1987). Stock return anomalies and the tests of the APT. *Journal of Finance, 42*(5), 1213–1224.
Hamao, Y. (1988). An empirical examination of the arbitrage pricing theory using Japanese data. *Japan and the World Economy, 1*(1), 45–61.
Hamada, R.S. (1969). Portfolio analysis, market equilibrium and corporation finance. *Journal of Finance, 24*(1), 13–31.
Hamada, R.S. (1972). The effects of the firm's capital structure on the systematic risk of common stock, *Journal of Finance, 27*(2), 435–52.
Hamilton, W.P. (1922). *The Stock Market Barometer*. New York, NY and London, UK: Harper & Brothers.
Hansen, L.P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica, 50*(4), 1029–1054.
Hansen, L.P., & Jagannathan, R. (1997). Assessing specification errors in stochastic discount factor models. *Journal of Finance, 52*(2), 557–590.

Hansen, L.P., & Singleton, K. (1982). Generalized instrumental variables estimation of nonlinear rational expectation models. *Econometrica, 50*(5), 1269–1288.

Hansen, L.P., & Singleton, K. (1983). Stochastic consumption, risk aversion, and the temporal behavior of asset returns. *Journal of Political Economy, 91*(2), 249–264.

Heaney, R., Koh, S., & Lan, Y. (2016), Australian firm characteristics and the cross–section variation in equity returns. *Pacific–Basin Finance Journal*, *37*, 104–115. DOI: 10.1016/j.pacfin.2016.03.001.

Heston, L.S., Rouwenhorst, K.G., & Wessels, R.E. (1999). The role of beta and size in the cross–section of European stock returns. *European Financial Management, 5*(1), 9–27.

Hicks, J.R. (1935). A suggestion of simplifying the theory of money. *Econometrica, 2*(5), 1–19.

Hirshleifer, D.A., Hou, K., Teoh, S.H., & Zhang, Y. (2004). Do investors overvalue firms with bloated balance sheets? *Journal of Accounting and Economics, 38*(1), 297–331.

Ho, C., & Hung, C. (2009). Investor sentiment as conditioning information in asset pricing. *Journal of Banking and Finance, 33*(5), 892–903.

Homma, M. (1755). The fountain of gold — the three monkey record of money.

Hong, D., Lee, C.M.C., & Swaminathan, B. (2003). *Earnings Momentum in International Markets* (SSRN Working Paper).

Hong, H., Lim, T., & Stein, J.C. (2000). Bad news travels slowly: Size, analyst coverage, and the profitability of momentum strategies. *Journal of Finance, 55*(1), 265–295.

Hong, H., & Stein, J.C. (1999). A unified theory of underreaction, momentum trading, and overreaction in asset markets. *Journal of Finance, 54*(6), 2143–2184.

Horowitz J.L., Loughran T., & Savin N.E. (2000a). The Disappearing Size Effect. *Research in Economics, 54*(1), 83–100.

Horowitz J.L., Loughran T., & Savin N.E. (2000b). Three analyses of the firm size premium. *Journal of Empirical Finance, 7*(2), 143–153.

Hou, K., Xue C., & Zhang L. (2015). Digesting anomalies: An investment approach. *Review of Financial Studies, 28*(3), 650–705.

Hou, K., Xue C., & Zhang L. (2016). *A comparison of new factor models*. Working paper.

Houge, T., & Loughran, T. (2006). Do investors capture the value premium? *Financial Management, 35*(2), 5–19.

Hsu, P.–H., Hsu, Y.–C., & Kuan, C.–M. (2010). Testing the predictive ability of technical analysis using a new stepwise test without data snooping bias. *Journal of Empirical Finance, 17*(3), 471–484.

Hsu, P.–H., & Kuan, C.–M. (2005). Re–examining the profitability of technical analysis with data snooping checks. *Journal of Financial Econometrics, 3*(4), 606–628.

Hudson, R., Dempsey, M., & Keasey, K. (1996). A note on the weak form efficiency of capital markets: The application of simple technical trading rules to UK stock prices – 1935 to 1964. *Journal of Banking & Finance, 20*(6), 1121–1132.

Hudson, Y., & Green, C.J. (2015). Is investor sentiment contagious? International sentiment and UK equity returns. *Journal of Behavioral and Experimental Finance, 5*, 46–59.

Huij, J., & Verbeek, M. (2009). On the use of multifactor models to evaluate mutual fund performance. *Financial Management, 38*(1), 75–102.

Ingersoll, J.E. (1984). Some results in the theory of arbitrage pricing. *Journal of Finance, 39*(4), 1021–1039.

Israel, R., & Moskowitz, T.J. (2013). The role of shorting, firm size, and time on market anomalies. *Journal of Financial Economics, 108*(2), 275–301.

Iqbal, J., & Haider, A. (2005). Arbitrage pricing theory: Evidence from an emerging market. *The Lahore Journal of Economics, 1*(10), 123–139.

Irwin, S.H., & Uhrig, J.W. (1984). Do technical analysts have holes in their shoes?, *Review of Research in Futures Markets, 3*(3), 264–277.

Jagadeesh, N., & Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance, 48*(1), 65–91.

Jagadeesh, N., & Titman, S. (2001). Profitability of momentum strategies: An evaluation of alternative explanations. *Journal of Finance, 56*(2), 699–720.

James, F.E.Jr. (1968). Monthly moving averages – an effective investment tool? *Journal of Financial and Quantitative Analysis, 3*(3), 315–326.

Jankowitsch, R., Nashikkar, A., & Subrahmanyam, M.G. (2011). Price dispersion in OTC markets: A new measure of liquidity. *Journal of Banking and Finance, 35*(2), 343–357.

Jansen, W.J., & Nahuic, J.N. (2003). The stock market and consumer confidence: European evidence. *Economics Letters, 79*(1), 89–98.

Jensen, M.C. (1968). Random walks: Reality or myth: Comment. *Financial Analysts Journal, 23*(6), 77–85.

Jensen, M.C. (1968). The performance of mutual funds in the period 1945–1964. *Journal of Finance, 23*(2), 389–416.
Jensen, M.C., & Benington, G.A. (1970). Random walks and technical theories: Some additional evidence. *Journal of Finance, 25*(2), 469–482.
Jiang, G., Lee, C.M.C., & Zhang, G.Y. (2005). Information uncertainty and expected returns. *Review of Accounting Studies, 10*(2), 185–221.
Jones, C.S. (2001). Extracting factors from heteroskedastic asset returns. *Journal of Financial Economics, 62*(2), 293–325.
Jorion, P. (1994). Mean–Variance Analysis of Currency Overlays. *Financial Analysts Journal, 50*(3), 48–56.
Kahneman, D., & Tversky, A.N. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47*(2), 263–291.
Kaiser, H.F. (1960). The application of electronic computers to factor analysis. *Educational and Psychological Measurement, 20*, 141–151.
Kan, R., Robotti, C., & Shanken, J. (2010). *Pricing model performance and the two–pass cross–sectional regression methodology*. Unpublished Manuscript.
Kandel, S. (1984). On the exclusion of assets from tests of the mean variance efficiency of the market portfolio. *Journal of Finance, 39*(1), 63–75.
Kaplanski, G. (2004). Traditional Beta, Downside Risk Beta and Market Risk Premiums. *Quarterly Reviews of Economics and Finance, 44*(5), 636–653.
Kausar, A., & Taffler, R. (2006). *Testing behavioural finance models of market under and overreaction: Do they really work?* (University of Edinburgh Working Paper).
Keene, M., & Peterson, D. (2007). The important of liquidity as a factor in asset pricing. *Journal of Financial Research, 30*(1), 91–109.
Keim, D.B. (1983). Size−related anomalies and stock return seasonality: further empirical evidence. *Journal of Financial Economics, 12*(1), 13−32.
Keim, D.B., & Stambaugh, R.F. (1986). Predicting returns in the stock and bond markets. *Journal of Financial Economics, 17*(2), 357−390.
Kendall, M.G. (1953). The analysis of economic time−series−part I: Prices. *Journal of the Royal Statistical Society, Series A*, *116*(1), 11–34.
Keynes, J.M. (1936). *The general theory of employment, interest and money*. London, UK: Macmillan.
Kho, B.–C. (1996). Time–varying risk premia, volatility, and technical trading rule profits: Evidence from foreign currency futures markets. *Journal of Financial Economics, 41*(2), 249–290.
Kim, T., & Ha, N. (2010). *Investor sentiment and market anomalies*. (SSRN Working Paper).
Kim, S.–H., & Kim D. (2012). Investor sentiment from internet message postings and the predictability of stock returns. *Journal of Economic Behavior & Organization, 107*(1), 708–729.
Kolmogorov, A., N., 1931, Über die analytischen methoden in der wahrscheinlichkeitsrechnung (on analytical methods in the theory of probability). *Mathematische Annalen*, *104*(1), 415–458.
Kothari, S.P., Lewellen, L., & Warner, J.B. (2006). Stock returns, aggregate earnings surprises, and behavioral finance. *Journal of Financial Economics, 79*(3), 537–568.
Kothari, S.P., & Shanken, J. (1995). In defense of beta. *Journal of Applied Corporate Finance, 8*(1), 53–58.
Kothari, S.P., Shanken, J., & Sloan, R.P. (1995). Another look at the cross–section of expected stock returns, *Journal of Finance, 50*(1), 185–224.
Kroll, Y., Levy, H., & Rapoport, A. (1988). Experimental tests of the separation theorem and the capital asset pricing model. *American Economic Review, 78*(3), 500–518.
Kuang, P., Schröder, M., & Wang, Q. (2014). Illusory profitability of technical analysis in emerging foreign exchange markets. *International Journal of Forecasting, 30*(2), 192–205.
Kumar, M., & Sehgal, S. (2004). Company characteristics and common stock returns: The Indian experience. *Investment Management and Financial Innovations, 4*, 89–99.
Lagoarde–Segot, T. (2009). Financial reforms and time–varying microstructures in emerging equity markets. *Journal of Banking and Finance, 33*(10), 1755–1769.
Lakonishok, J., Shleifer, A., & Vishny, R. (1994). Contrarian investment, extrapolation, and risk. *Journal of Finance, 49*(5), 1541–1578.
Lamoureux, C.G., & Sanger, G.C. (1989). Firm size and turn–of–the–year effects in the OTC/NASDAQ market. *Journal of Finance, 44*(5), 1219–1245.
La Porta, R. (1996). Expectations and the cross–section of stock returns. *Journal of Finance, 51*(5), 1715–1742.
Larson, A.B. (1960). Measurement of a random process in futures prices. *Food Research Institute Studies, 1*(3), 313–324.

Lease, R.C., Lewellen, W.G., & Schlarbaum, G.G. (1974). The individual investor: Attributes and attitudes. *Journal of Finance, 29*(2), 413– 433.
Leavens, D.H. (1945). Diversification of investments. *Trusts and Estates, 80*(5), 469–473.
LeBaron, B. (1999). Technical trading rule profitability and foreign exchange intervention. *Journal of International Economics, 49*(1), 125–143.
Lee, C.I., Gleason, K.C., & Mathur, I. (2001). Trading rule profits in Latin American currency spot rates. *International Review of Financial Analysis, 10*(2), 135–156.
Lee, C.I., Mathur, I. (1996a). Trading rule profits in European currency spot cross–rates. *Journal of Banking and Finance*, *20*(5), 949–962.
Lee, C.I., & Mathur, I. (1996b). A comprehensive look at the efficiency of technical trading rules applied to cross–rates. *European Journal of Finance, 2*, 389–411.
Lee, C.I., Pan, M., Liu, Y.A. (2001). On market efficiency of Asian foreign exchange rates: Evidence from a joint variance ratio test and technical trading rules. *Journal of International Financial Markets, Institutions and Money, 11*(2), 199–214.
Lee, C.M.C., Shleifer, A., & Thaler, R.H. (1991). Investor sentiment and the closed–end fund puzzle. *Journal of Finance, 46*(1), 75–109.
Lee, W.Y., Jiang, C.X., & Indro, D.C. (2002). Stock market volatility, excess returns, and the role of investor sentiment. *Journal of Banking & Finance, 26*(12), 2277–2299.
Lehmann, B.N., & Modest, D.M. (1988). The empirical foundations of the arbitrage pricing theory. *Journal of Financial Economics, 21*(2), 213–254.
Lemmon, M., & Portniaguina, E. (2006). Consumer confidence and asset prices: Some empirical evidence. *The Review of Financial Studies, 19*(4), 1499–1529.
Leigh, W., Modani, N., Purvis, R., & Roberts, T. (2002). Stock market trading rule discovery using technical charting heuristics. *Expert Systems with Applications, 23*(2), 155–159.
Leigh, W., Paz, N., & Purvis, R. (2002). Market timing: A test of a charting heuristic. *Economic Letters,* 77(1), 55–63.
LeRoy, S.F., & Porter, R. (1981). The present value relation: Tests based on variance bounds. *Econometrica, 49*(3), 555–577.
Levich, R.M., & Thomas, L.R., III. (1993). The significance of technical trading–rule profits in the foreign exchange market: A bootstrap approach. *Journal of International Money and Finance, 12*(5), 451–474.
Levy, R.A. (1967a). Random walks: Reality or myth. *Financial Analysts Journal, 23*(6), 69–77.
Levy, R.A. (1967b). Relative strength as a criterion for investment selection. *Journal of Finance, 22*(4), 595–610.
Levy, R.A. (1974). Beta coefficients as predictors of returns. *Financial Analysts Journal, 30*(1), 61–69.
Lewellen, J. (1999). The time series relations among expected return, risk, and book–to–market. *Journal of Financial Economics, 54*(1), 5–43.
Li, X., Brooks, C., & Miffre J. (2009). The value premium and time–varying volatility. *Journal of Business Finance and Accounting, 36*(9–10), 1252–1272.
Liao, T., Huang, C., & Wu, C. (2011). Do fund managers herd to counter investor sentiment?? *Journal of Business Research*, 64(2), 207–212.
Lintner, L. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics, 47*(1), 13–37.
Lin, S., & Rassenti, S. (2008). *Are under– and over–reaction the same matter? A price inertia based account* (Chapman University Economic Science Institute Working Paper Series, No. 05/08).
Linnainmaa, J.T., & Roberts, M.R. (2016). *The history of the cross section of stock returns* (NBER working paper, No. 22894).
Litzenberger, R., & Ramaswamy, K. (1979). The effects of personal taxes and dividends on capital asset prices: Theory and empirical evidence. *Journal of Financial Economics, 7*(2), 163–195.
Liu, W. (2006). A liquidity–augmented capital asset pricing model. *Journal of Financial Economics, 82*(3), 631–671.
Lo, A.W. (2004). The adaptive market hypothesis: Market efficiency from an evolutionary perspective. *Journal of Portfolio Management, 30*(5), 15–29.
Lo, A.W., Mamaysky, H., & Wang, J. (2000). Foundations of technical analysis: computational algorithms, statistical inference, and empirical implementation. *Journal of Finance, 55*(4), 1705–1765.
Logue, D.E., & Sweeney, R.J. (1977). White noise in imperfect markets: The case of the franc/dollar exchange rate. *Journal of Finance, 32*(3), 761–768.
Lopes, L. (1987). Between hope and fear: The psychology of risk. *Advances in Experimental Social Psychology, 20*(1), 255–295.

Lorenz, E.N. (1963), Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences, 20*(2), 130–141.
Loughran, T., & McDonald, B. (2011). When is a liability not a liability? Textual analysis, dictionaries, and 10–Ks. *Journal of Finance, 66*(1), 35–65.
Lucas, R.E. (1978). Asset prices in an exchange economy. *Econometrica, 46*(6), 1429–1445.
Lucke, B. (2003). Are technical trading rules profitable? Evidence for head–and–shoulder rules. *Applied Economics, 35*(1), 33–40.
Lukac, L.P., & Brorsen, B.W. (1989). The usefulness of historical data in selecting parameters for technical trading systems. *Journal of Futures Markets, 9*(1), 55–65.
Lukac, L.P., & Brorsen, B.W. (1990). A comprehensive test of futures market disequilibrium. *Financial Review, 25*(4), 593–622.
Maillet, B., & Michel, T. (2000). Further insights on the puzzle of technical analysis profitability. *European Journal of Finance, 6*(2), 196–224.
Malkiel B.G. (1973). *A Random Walk Down Wall Street*. USA: W. W. Norton & Company.
Mandelbrot, B. (1963). The Variation of Certain Speculative Prices. *Journal of Business, 36*(4), 394–419.
Mankiw G.N., & Shapiro, M.D. (1986). Risk and return: consumption beta versus market beta. *Review of Economics and Statistics, 68*(3), 452–459.
Markov, A.A. (1907). Extension of the limit theorems of probability theory to a sum of variables connected in a chain. In R., Howard. *Dynamic Probabilistic Systems, Volume 1: Markov Chains* (App. B). New York, NY: John Wiley & Sons, 1971.
Markowitz, H.M. (1952). Portfolio selection. *Journal of Finance*, *7*(1), 77–91.
Markowitz, H.M. (1952b). The utility of wealth. *Journal of Political Economy, 60*(2), 151–158.
Markowitz, H.M. (1959). Portfolio selection: Efficient diversification of investments. New York, NY: Yale University Press.
Markowitz, H.M. (1999). The early history of portfolio theory: 1600–1960. *Financial Analysts Journal, 55*(4), 5–16.
Markowitz, H.M. (Ed.). (2008). *Harry Markowitz: Selected works*. World Scientific — Nobel Laureate Series, 1: World Scientific Publishing Co. Pte. https://doi.org/10.1142/6967.
Marshall, A. (1890). *Principles of Economics*, *1*. London, UK: Macmillan and Co.
Marshall, B.R., Qian, S., & Young, M. (2009). Is technical analysis profitable on US stocks with certain size, liquidity or industry characteristics? *Applied Financial Economics, 19*(15), 1213–1221.
Martell, T.F. (1976). Adaptive trading rules for commodity future. *Omega, 4*(4), 407–416.
Martell, T.F., & Philippatos, G.C. (1974). Adaptation, information, and dependence in commodity markets. *Journal of Finance, 29*(2), 493–98.
Maslow, A.H. (1943), A theory of human motivation. *Psychological Review, 50*(4), 370–396.
McCallum, A.K. (1996). Bow: A toolkit for statistical language modeling, text Retrieval, classification and clustering. Carnegie–Mellon University. Pittsburgh, PA: School of Computer Science. http://www.cs.cmu.edu/mccallum/bow.
Merton, R.C. (1973). An intertemporal capital asset pricing model. *Econometrica, 41*(5), 867–887.
Merton, R.C. (1973). Theory of rational option pricing. *Bell Journal of Economics and Management Science, 4*(1), 141–183. (= Chapter 8 of Merton (1992)).
Merton, R.C. (1980). On estimating the expected return on the market: An exploratory investigation. *Journal of Financial Economics, 8*(4), 323–361.
Mian, M.G., & Sankaraguruswamy, S. (2008). *Investor Sentiment and Stock Market Response to Corporate News* (National Singapore University, Working Paper).
Middleton, L.P., & Satchell, S.E. (2001). Deriving the APT when the number of factors is unknown. *Quantitative finance, 1*(5), 502–508.
Milgrom, P.M. (1981). Good news and bad news. *Bell Journal of Economics, 12*(2), 379–391.
Mill, J.S. (1836). On the definition of political economy, and on the method of investigation proper to it. In D.M., Hausman (Ed.). *The philosophy of economics: An anthology* (3rd ed., pp. 41–58). Cambridge, UK: Cambridge University Press. https://doi.org/10.1017/CBO9780511819025.003.
Miller, M., & Scholes, M. (1972). Rate of return in relation to risk: A reexamination of some recent findings. In M.C., Jensen (Ed.). *Studies in the Theory of Capital Markets* (pp. 47–78). New York, NY: Praeger.
Miller, M., & Scholes, M. (1982). Dividend and taxes: Some empirical evidence. *Journal of Political Economy, 90*(6), 1118–1141.
Mills, T.C. (1997). Technical analysis and the London stock exchange: Testing trading rules using the FT30. *International Journal of Finance and Economics, 2*(4), 319–331.
Modigliani, F., & Brumberg R. (1954). Utility analysis and the consumption function: An interpretation of cross–

section data. In K.K., Kurihara (Ed.). *Post–Keynesian Economics* (pp. 388–436). New Brunswick, NJ: Rutgers University Press.

Morelli, D. (1999). Test of structural changes using factor analysis in equity returns. *Applied Economics Letters, 6*(4), 203–207.

Mossin, J. (1966). Equilibrium in a Capital Asset Market. *Econometrica, 34*(4), 768–783.

Mullainathan, S. (2001). *Thinking through categories*. MIT Economics Department Mimeo.

Neal, R., & Wheatley, S.M. (1998). Do measures of sentiment predict returns? *Journal of Financial Quantitative Analysis, 33*(4), 523–547.

Neely, C.J. (2002). The temporal pattern of trading rule returns and exchange rate intervention: Intervention does not generate technical trading profits. *Journal of International Economics, 58*(1), 211–232.

Neely, C.J., Papach, D.E., Tu, J., & Zhou, G. (2014). Forecasting the equity risk premium: The role of technical indicators. *Management Science, 60*(7), 1772–1791.

Neftci, S.N., & Policano, A.J. (1984). Can chartists outperform the market? Market efficiency tests for "technical analysis". *Journal of Futures Markets, 4*(4), 465–478.

Nguyen, N., Ulku, N., & Zhang, J. (2015). The Fama–French five factor model: Evidence from Vietnam. *New Zealand Finance Colloquium*, 1–29.

Nichol, E., & Dowling, M. (2014). Profitability and investment factors for UK asset pricing models. *Economics Letters*, *125*, 364–366.

Nison, S. (1991). Japanese Candlestick Charting Techniques, 15–18.

Novy–Marx, R. (2013). The other side of value: the gross profitability premium. *Journal of Financial Economics, 108*(1), 1–28.

O'Brien, M.A., Brailsford, T., & Gaunt, C. (2012). Size and book–to–market factors in Australia. *Australian Journal of Management, 37*(2), 261–281.

Officer, R. (1972). The Distribution of Stock Returns. *Journal of the American Statistical Association, 67*(340), 807–812.

Oldfield, G., Rogalski, R.J. (1981). Treasury bill factors and common stock returns. *Journal of Finance, 36*(2), 337–50.

Olson, D. (2004). Have trading rule profits in the currency markets declined over time? *Journal of Banking and Finance, 28*(1), 85–105.

Otoo, M.W. (1999). *Consumer Sentiment and the Stock Market* (Working Paper). Board of Governors of the Federal Reserve System.

Parisi, F., & Vasquez, A. (2000). Simple technical trading rules of stock returns: Evidence from 1987 to 1998 in Chile. *Emerging Markets Review, 1*(2), 152–164.

Park, C. & Irwin, S.H. (2004). *The profitability of technical analysis: A review* (AgMAS Project Research Report, No. 2004–04).

Pastor, L., & Stambaugh, R.F. (2003). Liquidity Risk and Expected Stock Returns. *Journal of Political Economy, 111*(3), 642–685.

Peters, E.E. (1991). *Chaos and order in the capital markets*. New York, NY: John Wiley & Sons.

Peters, E.E. (1994). *Fractal market analysis*. New York, NY: John Wiley & Sons.

Peterson, P.E., & Leuthold, R.M. (1982). Using mechanical trading systems to evaluate the weak form efficiency of futures markets. *Southern Journal of Agricultural Economics, 14*(1), 147–151.

Pettengill, G.N., Sundaram, S., & Mathur, I. (2002). Payment for risk: constant beta vs. dual–beta models. *The Financial Review, 27*, 123–136.

Pfiffelmann, M., Roger, T., & Bourachnikova, O. (2015). When Behavioral Portfolio Theory Meets Markowitz theory. *Economic Modelling, 53*, 419–435.

Piotroski, J.D. (2000). Value investing: The use of historical financial statement information to separate winners from losers. *Journal of Accounting Research, 98*, 1–41.

Poincaré, H. (1892). *New Methods of Celestial Mechanics, Vol.1,* (English trans. 1967), Reprinted With Endnotes by V. M. Alekseev and with an Introduction of Daniel F. Goroff, American Institute of Physics, New York, 1993.

Poincaré, H. (1893). *New Methods of Celestial Mechanics, Vol.2*, (English trans. 1967), Reprinted With Endnotes by V. M. Alekseev and with an Introduction of Daniel F. Goroff, American Institute of Physics, New York, 1993.

Poincaré, H. (1899). *New Methods of Celestial Mechanics, Vol.3*, (English trans. 1967), Reprinted With Endnotes by V. M. Alekseev and with an Introduction of Daniel F. Goroff, American Institute of Physics, New York, 1993.

Pontiff, J., & Woodgate, A. (2008). Share issuance and cross-sectional returns. *The Journal of Finance, 63*(2), 921–

945.
Poole, W. (1967). Speculative prices as random walks: An analysis of ten time series of flexible exchange rates. *Southern Economic Journal, 33*(2), 468–478.
Poon, S., & Taylor, S.J. (1991). Macroeconomic factors and the UK stock market. *Journal of Business Finance, 18*(5), 619–636.
Pruitt, S.W., & White, R.E. (1988). The CRISMA trading system: Who says technical analysis can't beat the market? *Journal of Portfolio Management, 14*(3), 55–58.
Qi, H. (2004). *An empirical study comparing the CAPM and the Fama–French 3–factor model* (Michigan Technological University Working Paper).
Qi, M., & Wu, Y. (2005). Technical trading–rule profitability, data snooping, and reality check: Evidence from the foreign exchange market. *Journal of Money, Credit and Banking, 38*(8), 2135–2158.
Qiu, L., & Welch, I. (2006). *Investor sentiment measures* (NBER Working Paper Series (SSRN ID 58941).
Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization, 3*(4), 323−343.
Quong, G., & Soudack, A. (1989). Volume–weighted RSI: Money flow. *Technical Analysis of Stocks & Commodities, 7*(3), 76–77.
Raj, M., Thurston, D. (1996). Effectiveness of Simple Technical Trading Rules in the Hong Kong Futures Markets. *Applied Economics Letters, 3*(1), 33–36.
Raza, H., Shah, S.A., & Malik, A.S. (2015). Day of the week anomaly and market efficiency: Evidence from KSE–Pakistan. *International Journal of Business and Social Science, 6*(9(1)), 69–75.
Regnault, J. (1863). *Calcul des chances et philosophie de la bourse*. Paris, France: Mallet–Bachelier and Castel.
Reinganum, M.R. (1981). A misspecification of capital asset pricing: Empirical anomalies based on earnings yields and market values. *Journal of Financial Economics, 9*(1), 19–46.
Reinganum, M.R. (1983). The anomalous stock market behavior of small firms in January: Empirical tests for tax−loss selling effects. *Journal of Financial Economics, 12*(1), 89−104.
Reisman, H. (1988). A general approach to the arbitrage pricing theory (APT). *Econometrica, 56*(2), 473–476.
Reisman, H. (2002). Some comments on the APT. *Quantitative Finance, 2*(6), 378–86.
Rhea, R. (1932). *The Dow Theory.* Reprinted by Horizon Publishing Company, 2015.
Rogers, P., & Securato, J.R. (2007). *Reward beta approach: A review* (Griffith University Working Paper). Dept. of Accounting, Finance and Economics.
Rogers, P., & Securato, J.R. (2009). Comparative study on the Brazilian market of the capital asset pricing model CAPM Fama and French 3factors model and reward beta approach English. *Associacao Nacional de Pos–Graduacao e Pesquisa em Administracao (ANPAD), 3*(1), 159–179.
Roll, R. (1970). *The behaviour of interest rates*. New York, NY: Basic Books.
Roll, R. (1977). A critique of the asset pricing theory's tests part I: On past and potential testability of the theory. *Journal of Financial Economics, 4*(2), 129–176.
Roll, R., & Ross, S.A. (1980). An empirical investigation of the arbitrage pricing theory. *Journal of Finance, 35*(5), 1073–1103.
Roll, R., & Ross, S.A. (1984). A critical reexamination of the empirical evidence on the arbitrage pricing theory: A reply. *Journal of Finance, 39*(2), 347–350.
Roll, R., & Ross, S.A. (1995). The arbitrage pricing theory approach to strategic portfolio planning. *Financial Analysts Journal, 51*(1), 122–131.
Rosenberg, B., Reid, K., & Lanstein, R. (1985). Persuasive evidence of market inefficiency. *Journal of Portfolio Management, 11*(3), 9–16.
Ross, S.A. (1976). The arbitrage theory of capital pricing. *Journal of Economic Theory, 13*(3), 341–360.
Ross, S.A. (2002). Neoclassical finance, alternative finance and the closed end fund puzzle. *European Financial Management, 8*(2), 129–137.
Ross, S.A. (2005). *Neoclassical finance*. Princeton, US: Princeton University Press.
Rowenhorst, K.G. (1998). Local return factors and turnover in emerging stock markets. *Journal of Finance, 54*(4), 1439–1464.
Roy, A.D. (1952). Safety first and the holding of assets. *Econometrica*, *20*(3), 431–449.
Rubinstein, M. (1976). The valuation of uncertain income streams and the pricing of options. *Bell Journal of Economics and Management Science, 7*(2), 407–425.
Sadaqat, M., & Butt, H. A. (2016). Modeling sentiment, temporal volatility and excess returns: Empirical evidence from segmented stock market. *Journal of Business and Economics, 8*(2), 202–228.
Samer AM, A.R., Abdullah, Al.Y., & Izz, E.N.A. (2010). Beta wins again: Case of four emerging markets. *Journal of Economic Cooperation and Development, 31*(1), 1–16.
Samuelson, P.A. (1938), A note on the pure theory of consumer's behavior. *Economica, New Series, 5*(17), 61–

71.
Samuelson, P.A. (1963). Risk and uncertainty: A fallacy of large numbers. *Scientia, 57*(6), 50–55.
Samuelson, P.A. (1965). Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review, 6*(2), 41–49.
Samuelson, P.A. (1998). Summing up on business cycles: Opening address. In J.C., Fuhrer & S., Schuh, *Beyond Shocks: What Causes Business Cycles* (pp. 4–6). Boston, USA: Federal Reserve Bank of Boston.
Sanders, D.R., Irwin, S.H., & Leuthold, R.M. (Eds.). (1997). *Noise Trader Sentiment and Futures Price Behavior: An Empirical Investigation*. Paper presented at the NCR–134 Conference on Applied Commodity Price Analysis, Forecasting and Market Risk Management (pp. 12–23). Chicago, IL.
Sapp, S. (2004). Are all central bank interventions created equal? An empirical investigation. *Journal of Banking and Finance, 28*(3), 443–474.
Sayim, M., Morris, P.D., & Rahman, H. (2013). The effect of us individual investor sentiment on industry specific stock returns and volatility. *Review of Behavioral Finance, 5*(1), 58–76.
Schabacker, R.W. (2005). *Technical Analysis and Stock Market Profits*, Introduced and Edited by Donald Mack.
Scheinkman, J.A., & Xiong, W. (2003). Overconfidence and speculative bubbles. *Journal of Political Economy, 111*(6), 1183–1220.
Schmeling, M. (2009). Investor sentiment and stock returns: Some international evidence. *Journal of Empirical Finance, 16*(3), 394–408.
Scholes, M., & Williams, J. (1977). Estimating betas from nonsynchronous data. *Journal of Financial Economics, 5*(3), 309–328.
Schwert, W.G. (2003). Anomalies and Market Efficiency. G.M., Constantinides, M., Harris, & R. Stulz (Eds.). *Handbook of the Economics of Finance* (pp. 937–972). North Holland: Elsevier Science B.V.
Scruggs, J.T. (1998). Resolving the puzzling intertemporal relation between the market risk premium and conditional market variance: A two–factor approach. *Journal of Finance, 53*(2), 575–603.
Segal, U. (1987). Some remarks on Quiggin's anticipated utility. *Journal of Economic Behavior and Organization, 8*(1), 145−154.
Segal, U. (1989). Anticipated utility: A measure representation approach. *Annals of Operations Research, 19*(1), 359−373.
Shanken, J. (1982). The arbitrage pricing theory: Is it testable? *Journal of Finance, 37*(5), 1129–1140.
Sharpe, W.F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance, 19*(3), 425–442.
Sharpe, W.F. (1966). Mutual fund performance. *Journal of Business, 39*(1–2), 119–138.
Sharpe, W.F. (1982). Factors in New York stock exchange security returns, 1931–1979. *Journal of Portfolio Management, 8*(4), 5–19.
Sharpe, W.F. (1994). The Sharpe ratio. *The Journal of Portfolio Management, 21*(1), 49–58.
Shefrin, H.M., & Thaler, R.H. (1988). The behavioral life–cycle hypothesis. *Economic Inquiry, 26*(4), 609–643.
Shefrin, H.M. & Statman M. (1994). Behavioral Capital Asset Pricing Theory. *Journal of Financial and Quantitative Analysis,* 29(3), 323–349.
Shefrin, H.M., & Statman M. (2000). Behavioral portfolio theory. *Journal of Financial and Quantitative Analysis, 35*(2), 127–51.
Shefrin, H.M. (2008). A behavioral approach to asset pricing. *Academic Press Advanced Finance* (2nd ed.). Burlington, MA: Elsevier Science Publishing.
Shiller, R.J. (1981). Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review, 71*(3), 421–436.
Shiller, R.J. (1981). Alternative tests of rational expectations models: The case of the term structure. *Journal of Econometrics*, *16*(1), 17–87.
Shiller, R.J. (2003). From efficient markets theory to behavioral finance. *Journal of Economic Perspectives, 17*(1), 83–104.
Shukla, R., & Trzcinka, C. (1990). Sequential tests of the arbitrage pricing theory: A comparison of principal components and maximum likelihood factors. *Journal of Finance, 45*(5), 1541–1564.
Shukla, R., & Trzcinka, C. (1991). Research on risk and return: Can measures of risk explain anything? *Journal of Portfolio Management, 17*(3), 15–21.
Silber, W.L. (1994). Technical trading: When it works and when it doesn't. *Journal of Derivatives, 1*(3), 39–44.
Simon, H.A. (1955). A Behavioral Model of Rational Choice. *The Quarterly Journal of Economics, 69*(1), 99–118.
Simon, H.A. (1957). *Models of man, social and rational: Mathematical essays on rational human behavior in a social setting*. New York, NY: John Wiley & Sons.
Skinner, D J., & Sloan, R.G. (2002). Earnings surprises, growth expectations, and stock returns or don't let an

earning torpedo sink your portfolio. *Review of Accounting Studies, 7*(2), 289–312.
Smidt, S. (1965b). A test of serial independence of price changes in soybean futures. *Food Research Institute Studies, 5*(2), 117–136.
Smith, A. (1757). *The theory of moral sentiments*. K. Haakonssen (Ed.). Cambridge, UK: Cambridge University Press.
Smith, A. (1776). *An inquiry into the nature and causes of the wealth of nations*. London, UK: W. Strahan; T.Cadell, Retrieved 10 March 2015, UK.
Smirlock, M., & Starks, L. (1986). Day–of–the–week and intraday effects in stock returns. *Journal of Financial Economics, 17*(1), 197–210.
Solt, M.E., & Statman, M. (1988). How useful is the sentiment index? *Financial Analysts Journal, 44*(5), 45–55.
Solt, M.E., & Swanson, P.J. (1981). On the efficiency of the markets for gold and silver. *Journal of Business, 54*(3), 453–478.
Sosvilla–Rivero, S., Andrada–Félix, J., & Fernández–Rodríguez, F. (2002). Further evidence on technical trade profitability and foreign exchange intervention. *Applied Economics Letters, 9*(12), 827–832.
Spiegel, M.I., & Wang, X. (2005). *Cross–sectional variation in stock returns: Liquidity and idiosyncratic risk* (Yale ICF Working Paper, No. 05–13).
Stambaugh, R.F. (1982). On the exclusion of assets from tests of the two–parameter model: a sensitivity analysis. *Journal of Financial Economics, 10*(3), 237–268.
Stambaugh, R.F., Yu, J., & Yuan, Y. (2012a). The short of it: Investor sentiment and anomalies. *Journal of Financial Economics, 104*(2), 288–302.
Stambaugh, R.F., Yu, J., & Yuan, Y. (2012b). *The long of it: Odds that investor sentiment spuriously predicts anomaly returns* (University of Pennsylvania Working Paper).
Statman, M., Thorley, S., & Vorkink, K. (2006). Investor overconfidence and trading volume. *Review of Financial Studies, 19*(4), 1531–1565.
Stattman, D. (1980). Book value and stock returns. *The Chicago MBA, Journal of Selected Papers, 4*, 25−45.
Stewart, B. (1949). *An Analysis of Speculative Trading in Grain Futures* (Technical Bulletin, No. 1001). US Department of Agriculture, Washington, D.C., USA.
Sullivan, R., Timmermann, A., & White, H. (1999). Data snooping, technical trading rule performance, and the bootstrap. *Journal of Finance, 54*(5), 1647–1691.
Sullivan, R., Timmermann, A., & White, H. (2000). Forecast evaluation with shared data sets. *International Journal of Forecasting, 19*(2), 217–227.
Sweeney, R.J. (1986). Beating the foreign exchange market. *Journal of finance, 41*(1), 163–182.
Sweeney, R.J. (1988). Some new filter rule tests: Methods and results. *Journal of Financial and Quantitative Analysis, 23*(3), 285–300.
Sweeney, R.J., & Surajaras, P. (1989). The Stability of Speculative Profits in the Foreign Exchanges. In R.M.C., Guimarães, B.G., Kingsman & S.J. Taylor (Eds.). *A Reappraisal of the Efficiency of Financial Markets* (pp. 547–576). Springer Berlin Heidelberg.
Szakmary, A.C., & Mathur, I. (1997). Central bank intervention and trading rule profits in foreign exchange markets. *Journal of International Money and Finance, 16*(4), 513–535.
Szyszka, A. (2007). *Behavioral asset pricing (in Polish)*. Poznań, Poland: Poznan University of Economics and Business Press.
Szyszka, A. (2009). Generalized behavioral asset pricing model. *IFCAI Journal of Behavioral Finance, 6*(1), 7–25.
Taylor, N. (2014). The rise and fall of technical trading rule success. *Journal of Banking & Finance, 40*(17), 286–302.
Taylor, S.J. (1992). Rewards available to currency futures speculators: Compensation for risk or evidence of inefficient pricing? *Economic Record, 68*(S1), 105–116.
Taylor, S.J. (1994). Trading futures using a channel rule: A study of the predictive power of technical analysis with currency examples. *Journal of Futures Markets, 14*(2), 215–235.
Tetlock, P.C. (2007). Giving content to investor sentiment: The role of media in the stock market. *Journal of Finance, 62*(3), 1139–1168.
Tetlock, P.C., Saar–Tsechansky, M., & Macskassy S. (2008). More than words: Quantifying language to measure firms' fundamentals. *Journal of Finance, 63*(3), 1437–1467.
Thaler, R.H. (1980). Toward a positive theory of consumer choice. *Journal of Economic Behavior & Organization, 1*(1), 39–60.
Thaler, R.H. (1985). Mental accounting and consumer choice. *Marketing Science, 4*(3), 199–214.
Titman, S., Wei, K.C., & Xie, F. (2003). Capital investments and stock returns (NBER Working Paper, No. w9951). National Bureau of Economic Research.

Tobin, J. (1958). Liquidity preferences as behaviour toward risk. *Review of Economic Studies, 25*(2), 65–86.
Treynor, J.L. (1961). *Market value, time, and risk*. Unpublished Manuscript.
Treynor, J.L. (1962). Toward a theory of market value of risky assets. Unpublished manuscript, A Final Version Was Published in 1999. In R.A. Korajczyk (Ed.). *Asset pricing and portfolio performance: Models, strategy and performance metrics* (pp. 15–22). London, UK: Risk Books.
Treynor, J.L. (1966). How to rate management investment funds. *Harvard Business Review, 43*(1), 63–75.
Trzcinka, C. (1986). On the number of factors in the arbitrage pricing model. *Journal of Finance, 41*(2), 347–368.
Tumarkin, R. (2002). Internet message board activity and market efficiency: A case study of the internet service sector using ragingbull.com. *Financial Markets Institutions and Instruments, 11*(4), 313–335.
Tumarkin, R., & Whitelaw, R.F. (2001). News or noise? Internet postings and stock prices. *Financial Analysts Journal, 57*(3), 41–51.
Tursoy, T., Gunsel, T., & Rjoub, H. (2008). Macroeconomic factors, the APT and the Istanbul stock market. *International Research Journal of Finance and Economics, 56*(22), 49–57.
Tversky, A.N., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science, New Series, 185*(4157), 1124–1131.
Uygur, U., & Taş, O. (2012). Modeling the effects of investor sentiment and conditional volatility in international stock markets. *Journal of Applied Finance & Banking, Sciencepress Ltd, 2*(5), 239–260.
Van Horne, J.C., & Parker, G.G.C. (1967). The random–walk theory: An empirical test. *Financial Analysts Journal, 23*(6), 87–92.
Van Horne, J.C. & Parker, G.G.C. (1968). Technical trading rules: A comment. *Financial Analysts Journal, 24*(4), 128–132.
Vidojevic, M., Hanauer, M.X., & Blitz D. (2017). *The idiosyncratic momentum anomaly* (SSRN Working Paper).
Von Neumann, J., & Morgenstern, O. (1944, 1953). *Theory of Games and Economic Behavior* (2nd ed.). Princeton, US: Princeton University Press.
Wang, J.–L. (1994). A model of competitive stock trading volume. *Journal of Political Economy, 102*(1), 127–168.
Wang, J.–L., & Chan, S.–H. (2009). Trading rule discovery in the US stock market: An empirical study. *Expert Systems with Applications, 36*(3), 5450–5455.
Wang, Y.H., Keswani, A., & Taylor, S.J. (2006). The relationships between sentiment, returns and volatility. *International Journal of Forecasting, 22*(1), 109–123.
Weiss, K. (1989). The post–offering price performance of closed–end funds. *Financial Management, 18*(3), 57–67.
Wheatley, S. (1988). Some tests of international equity integration. *Journal of Financial Economics, 21*(2), 177–212.
Wilder, J.W. (1978). *New concepts in technical trading systems*, Trend Research.
Williams, J.B. (1938). *The Theory of Investment Value*. Cambridge, MA: Harvard University Press.
Working, H. (1958). A theory of anticipated prices. *The American Economic Review, 48*(2), 188–199.
Wu, H. (2011). The value and size effect – are there firm–specific–risks in China's domestic stock markets? *International Journal of Economics and Finance, 3*(3), 26–37.
Wysocki, P.D. (1998, 1999). *Cheap talk on the web: The determinants of postings on stock message boards* (University of Michigan Business School Working Paper, No. 98025).
Xiuqing, J. (2008). Is it under– or over–reaction? An empirical investigation. *Asia–Pacific Journal of Financial Studies, 37*(2), 185–216.
Yaari, M.E. (1987). The dual theory of choice under risk. *Econometrica, 55*(1), 95–115.
Yang, C., & Gao, B. (2014). The term structure of sentiment effect in stock index futures market. *The North American Journal of Economics and Finance, 30*, 171–182.
Yang, C., & Zhang, R. (2014). Does mixed–frequency investor sentiment impact stock returns? Based on the empirical study of MIDAS regression model. *Applied Economics, 46*(9), 966–972.
Yang, C., & Zhou, L. (2015). Investor trading behavior, investor sentiment and asset prices. *The North American Journal of Economics and Finance, 34*, 42–62.
Yu, H., Nartea, G., Gan, C., & Yao, L. (2013). Predictive ability and profitability of simple technical trading rules: Recent evidence from southeast Asian stock markets. *International Review of Economics and Finance, 25*(1), 356–371.
Zapranis, A., & Tsinaslanidis, P.E. (2012). A novel, rule–based technical pattern identification mechanism: Identifying and evaluating saucers and resistant levels in the US stock market. *Expert Systems with Applications, 39*(7), 6301–6308.
Zhang, X.F. (2006). Information uncertainty and stock returns. *Journal of Finance, 61*(1), 105–137.
Zhou, L., & Zhang, D. (2016). An empirical study of the effectiveness of China's stock market – based on the Fama–

French five–factor model. *Manager' Journal, 5*.
Zouaoui, M., Nouyrigat, G., & Beer, F. (2011). How does Investor Sentiment Affect Stock Market Crises? Evidence from Panel Data. *Financial Review, 46*(4), 723–747.

List of graphs

List of figures

List of tables

Annex

Table 3.1 List of companies used in the sample for US and Israeli markets

Israeli market – 14 companies from the TL35 index former TL25

NAME	INDUSTRY
AVNER L	OIL & GAS
BAZEQ	COMMUNICATION
DELEK DRILL	OIL & GAS
DELEK GROUP	ENERGY
DISCOUNT	BANKING
ELBIT SYSTEMS	DEFENCE
ICL	CHEMESTRY
ISRAEL CORP	CHEMESTRY
ISRAMCO	OIL & GAS
LEUMI	BANKING
MIZRAHI	BANKING
NICE	MANAGEMENT
POALIM	BANKING
TEVA	PHARMACOLOGY

US market – 50 companies from the NASDAQ100 index

NAME	INDUSTRY
Adobe Systems Incorporated (ADBE)	Software - Application
Akamai Technologies, Inc. (AKAM)	Internet Information Providers
Alexion Pharmaceuticals, Inc. (ALXN)	Biotechnology
Amazon.com, Inc. (AMZN)	Specialty Retail
Amgen Inc. (AMGN)	Biotechnology
Analog Devices, Inc. (ADI)	Semiconductor - Integrated Circuits
Apple Inc. (AAPL)	Electronic Equipment
Autodesk, Inc. (ADSK)	Software - Application
Automatic Data Processing, Inc. (ADP)	Business Services
Biogen Inc. (BIIB)	Biotechnology
BioMarin Pharmaceutical Inc. (BMRN)	Biotechnology
CA, Inc. (CA)	Software - Infrastructure
Cerner Corporation (CERN)	Health Information Services
Check Point Software Technologies Ltd. (CHKP)	Security Software & Services
Cintas Corporation (CTAS)	Business Services
Cisco Systems, Inc. (CSCO)	Communication Equipment
Cognizant Technology Solutions Corporation (CTSH)	Information Technology Services
Comcast Corporation (CMCSA)	Entertainment - Diversified
Costco Wholesale Corporation (COST)	Discount, Variety Stores
CSX Corporation (CSX)	Railroads
DENTSPLY SIRONA Inc. (XRAY)	Medical Instruments & Supplies
DISH Network Corporation (DISH)	Semiconductor - Broad Line
Dollar Tree, Inc. (DLTR)	Discount, Variety Stores
eBay Inc. (EBAY)	Specialty Retail, Other
Electronic Arts Inc. (EA)	Electronic Gaming & Multimedia
Express Scripts Holding Company (ESRX)	Health Care Plans
Fastenal Company (FAST)	Industrial Distribution
Fiserv, Inc. (FISV)	Business Services
Hasbro, Inc. (HAS)	Leisure

Hologic, Inc. (HOLX)	Medical Appliances & Equipment
IDEXX Laboratories, Inc. (IDXX)	Diagnostic Substances
J.B. Hunt Transport Services, Inc. (JBHT)	Trucking
KLA-Tencor Corporation (KLAC)	Semiconductor Equipment & Materials
Lam Research Corporation (LRCX)	Semiconductor Equipment & Materials
Marriott International, Inc. (MAR)	Lodging
Mattel, Inc. (MAT)	Leisure
Microsoft Corporation (MSFT)	Business Software & Services
Mylan N.V. (MYL)	Drugs - Generic
NVIDIA Corporation (NVDA)	Semiconductor - Specialized
O'Reilly Automotive, Inc. (ORLY)	Auto Parts Stores
PACCAR Inc (PCAR)	Truck Manufacturing
Paychex, Inc. (PAYX)	Staffing & Outsourcing Services
Priceline Group Inc. (PCLN)	Business Services
Ross Stores, Inc. (ROST)	Apparel Stores
Skyworks Solutions, Inc. (SWKS)	Semiconductor - Integrated Circuits
Symantec Corporation (SYMC)	Security Software & Services
Tractor Supply Company (TSCO)	Specialty Retail, Other
Vertex Pharmaceuticals Incorporated (VRTX)	Biotechnology
Walgreens Boots Alliance, Inc. (WBA)	Drug Stores
Western Digital Corporation (WDC)	Data Storage Devices

Table 3.2b Average values of original/direct technical indicators coefficients per level of significance

		Const.	SMA_t^{110}	SMA_t^{120}	SMA_t^{150}	SMA_t^{510}	SMA_t^{520}	SMA_t^{550}	$MAROC_t^{510}$	$MAROC_t^{520}$	$MAROC_t^{550}$	$MA110_t^{OBV}$	$MA120_t^{OBV}$	$MA150_t^{OBV}$	$MA510_t^{OBV}$	$MA520_t^{OBV}$	$MA550_t^{OBV}$	R^2	R_{adj}^2
US	Panel A	0.00825	0.00597	0.00641	0.00724	-0.00684	-0.00379	-0.00483	-0.00783	-0.00724	-0.00648	0.00438	0.00353	0.00460	-0.00495	-0.00295	-0.00384	0.29198	0.28933
	Panel B	0.01057	0.00766	0.00830	0.00967	-0.00882	-0.00672	-0.00722	-0.01013	-0.00936	-0.00769	0.00625	0.00801	0.00801	-0.00693	-0.00530	-0.00752	0.31191	0.30359
	Panel C	0.00864	0.00679	0.00648	0.00789	-0.00694	-0.00450	-0.00617	-0.00785	-0.00731	-0.00751	0.00515	0.00515	0.00615	-0.00534	-0.00500	-0.00571	0.30490	0.29651
	Panel D	0.00630	0.00426	0.00535	0.00596	-0.00533	-0.00370	-0.00455	-0.00585	-0.00532	-0.00482	0.00369	0.00385	0.00430	-0.00405	-0.00346	-0.00414	0.32600	0.31923
Israel	Panel A	0.00738	0.00574	0.00503	0.00512	-0.00635	-0.00313	-0.00267	-0.00715	-0.00623	-0.00618	0.00352	0.00322	0.00338	-0.00427	-0.00260	-0.00266	0.28903	0.28630
	Panel B	0.00754	0.00564	0.00560	0.00633	-0.00650	-0.00391	-0.00475	-0.00681	-0.00649	-0.00620	0.00377	0.00510	0.00377	-0.00497	-0.00510	-0.00390	0.32167	0.31327
	Panel C	0.00838	0.00648	0.00619	0.00685	-0.00756	-0.00368	-0.00385	-0.00851	-0.00685	-0.00732	0.00593	0.00518	0.00592	-0.00573	-0.00387	-0.00580	0.29552	0.28680
	Panel D	0.00648	0.00554	0.00415	0.00429	-0.00523	-0.00294	-0.00241	-0.00639	-0.00540	-0.00527	0.00280	0.00219	0.00208	-0.00374	-0.00100	-0.00135	0.32631	0.31936

Source: Own analysis

Table 3.3b Average values of original/direct sentiment indicators coefficients per level of significance

		Const.	RSI_t	MFI_t	PLI_t	ATR_t	DVA_t	R^2	R_{adj}^2
US	Panel A	-0.00785	0.00053	-0.00017	-0.00024	1.20834	0.000000000014353	0.58414	0.58361
	Panel B	-0.01253	0.00069	-0.00024	-0.00029	1.52973	0.000000000042074	0.59246	0.59083
	Panel C	-0.00723	0.00051	-0.00018	-0.00027	1.18551	0.000000000018896	0.62357	0.62207
	Panel D	-0.00614	0.00038	-0.00013	-0.00017	1.25754	0.000000000018460	0.65523	0.65409
Israel	Panel A	-0.01223	0.00060	-0.00018	-0.00022	4.80165	0.000000000000497	0.50002	0.49938
	Panel B	-0.01178	0.00057	-0.00017	-0.00021	5.59126	-0.000000000019897	0.48253	0.48041
	Panel C	-0.01420	0.00066	-0.00020	-0.00024	4.46280	0.000000000000606	0.53072	0.52880
	Panel D	-0.01009	0.00056	-0.00017	-0.00021	5.79225	0.000000000000513	0.55183	0.55030

Source: Own analysis

Table 3.8c Average values of coefficients for predictors derived from PCA

		Const.	TECHNICAL INDICATORS				Const.	SENTIMENT INDICATORS		Const.	UNIFIED INDICATORS				
			$TA1_t$	$TA2_t$	$TA3_t$	$(TA4_t)$		$SI1_t$	$(SI2_t)$		$ALL1_t$	$ALL2_t$	$ALL3_t$	$ALL4_t$	$(ALL5_t)$
US	Panel A	0.00086	0.00215	-0.00584	-0.00574	-0.00249	0.00089	0.00532	-0.00072	0.00086	0.00212	-0.00722	-0.00718	-0.00086	-0.00030
	Panel B	0.00148	0.00264	-0.00707	-0.00704	-0.00430	0.00150	0.00664	-0.00048	0.00144	0.00262	-0.00871	-0.00962	-0.00133	-0.00024
	Panel C	0.00150	0.00223	-0.00614	-0.00565	-0.00415	0.00165	0.00553	0.00267	0.00130	0.00221	-0.00768	-0.00577	-0.00099	-0.00079
	Panel D	0.00087	0.00166	-0.00444	-0.00416	-0.00209	0.00089	0.00418	0.00259	0.00083	0.00165	-0.00552	-0.00494	-0.00073	0.00073
Israel	Panel A	0.00064	0.00190	-0.00535	-0.00261	-0.00463	0.00070	0.00478	-0.00330	0.00060	0.00191	-0.00639	-0.00225	-0.00475	0.00074
	Panel B	0.00101	0.00182	-0.00502	-0.00307	-0.00422	0.00101	0.00448	0.00562	0.00085	0.00184	-0.00594	-0.00352	-0.00470	0.00176
	Panel C	0.00158	0.00230	-0.00588	-0.00486	-0.00538	0.00158	0.00589	-0.00172	0.00157	0.00233	-0.00715	-0.00316	-0.00207	-0.00912
	Panel D	-0.00013	0.00163	-0.00491	-0.00269	-0.00345	-0.00038	0.00413	-0.00265	-0.00013	0.00163	-0.00579	-0.00188	-0.00367	-0.00052

Source: Own analysis

Table 3.11c Average values of coefficients for integrated technical model

		Const.	TECHNICAL INDICATORS				RES_t^{MKT} / RES_t^{TA}	RES_t^{SMB}	RES_t^{HML}	RES_t^{RMW}	RES_t^{CMA}
			$TA1_t$	$TA2_t$	$TA3_t$	$(TA4_t)$					
US	Panel A	0.00069	0.00215	-0.00584	-0.00574	-0.00249	0.84287	0.06401	-0.32887	-0.43301	-0.05787
	Panel B	0.00076	0.00264	-0.00707	-0.00704	-0.00430	0.84901	0.25393	-0.53018	-0.60905	-0.35356
	Panel C	0.00058	0.00223	-0.00614	-0.00565	-0.00415	0.86442	0.15078	-0.29847	-0.16536	0.16203
	Panel D	0.00073	0.00166	-0.00444	-0.00424	-0.00209	0.81852	0.01942	-0.33239	-0.10118	-0.15749
Israel	Panel A	0.00036	0.00190	-0.00535	-0.00261	-0.00463	0.80357	N/A	N/A	N/A	N/A
	Panel B	0.00042	0.00182	-0.00502	-0.00307	-0.00422	0.75707	N/A	N/A	N/A	N/A
	Panel C	0.00064	0.00230	-0.00588	-0.00486	-0.00538	0.83033	N/A	N/A	N/A	N/A
	Panel D	0.00006	0.00163	-0.00491	-0.00269	-0.00345	0.83737	N/A	N/A	N/A	N/A

Source: Own analysis

Table 3.11d Average values of coefficients for integrated sentiment model

		Const.	SENTIMENT INDICATORS		RES_t^{MKT} / RES_t^{SI}	RES_t^{SMB}	RES_t^{HML}	RES_t^{RMW}	RES_t^{CMA}
			$SI1_t$	$(SI2_t)$					
US	Panel A	0.00086	0.00532	-0.00072	0.92132	0.10980	-0.37649	-0.40608	-0.05279
	Panel B	0.00141	0.00664	-0.00048	0.91707	0.24894	-0.51539	-0.54273	-0.14855
	Panel C	0.00123	0.00553	0.00267	0.89699	0.19492	-0.31954	-0.18869	0.05540
	Panel D	0.00082	0.00418	0.00259	0.90592	0.02660	-0.39901	-0.00746	-0.04381
Israel	Panel A	0.00060	0.00478	-0.00330	0.87834	N/A	N/A	N/A	N/A
	Panel B	0.00069	0.00448	0.00562	0.85383	N/A	N/A	N/A	N/A
	Panel C	0.00157	0.00589	-0.00172	0.88239	N/A	N/A	N/A	N/A
	Panel D	-0.00013	0.00413	-0.00265	0.86691	N/A	N/A	N/A	N/A

Source: Own analysis

Table 3.11e Average values of coefficients for integrated unified model

		Const.	UNIFIED INDICATORS					RES_t^{MKT} / RES_t^{ALL}	RES_t^{SMB}	RES_t^{HML}	RES_t^{RMW}	RES_t^{CMA}
			$ALL1_t$	$ALL2_t$	$ALL3_t$	$ALL4_t$	$(ALL5_t)$					
US	Panel A	0.00080	0.00212	-0.00722	-0.00718	-0.00086	-0.00030	0.67861	-0.01856	-0.26640	-0.36201	-0.10064
	Panel B	0.00140	0.00262	-0.00871	-0.00939	-0.00162	-0.00024	0.66525	0.10279	-0.43480	-0.53464	-0.36399
	Panel C	0.00113	0.00221	-0.00768	-0.00577	-0.00099	-0.00079	0.71747	0.05802	-0.23787	-0.08993	0.10765
	Panel D	0.00081	0.00164	-0.00551	-0.00492	-0.00064	0.00071	0.64354	0.00484	-0.24733	-0.07527	-0.16741
Israel	Panel A	0.00056	0.00191	-0.00639	-0.00211	-0.00443	0.00067	0.67530	N/A	N/A	N/A	N/A
	Panel B	0.00066	0.00184	-0.00594	-0.00352	-0.00470	0.00176	0.62999	N/A	N/A	N/A	N/A
	Panel C	0.00157	0.00233	-0.00715	-0.00285	-0.00192	-0.00912	0.69341	N/A	N/A	N/A	N/A
	Panel D	-0.00002	0.00163	-0.00579	-0.00188	-0.00367	-0.00052	0.68522	N/A	N/A	N/A	N/A

Source: Own analysis

Table 3.14b Average values of coefficients for integrated alternative model

	Const.	TECHNICAL INDICATORS				SENTIMENT INDICATORS		RES_t^{MKT} / RES_t^{TASI}	RES_t^{SMB}	RES_t^{HML}	RES_t^{RMW}	RES_t^{CMA}
		TA1	TA2	TA3	TA4	$SI1_t$	$(SI2_t)$					
US	0.00079	-0.00191	-0.00586	-0.00440	-0.00173	0.00803	-0.00046	0.75574	0.02454	-0.28689	-0.37632	-0.09288
Israel	0.00058	-0.00190	-0.00520	-0.00163	-0.00172	0.00736	-0.00291	0.70088	N/A	N/A	N/A	N/A

Source: Own analysis

Summary

For decades, the debates of the nature of the deviation from the fundamental price (*F*) is either rational or behavioral. Two main financial theories — normative and behavioral — exist side by side, describing the same financial phenomena of capital asset pricing by different explanations that even contradict each other. None of them has enough evidence to reject the competitive one, while each of them has sufficient evidence to support their own views. Both theories are good, but seems like not good enough. Otherwise, only one theory would give an appropriate description of the financial reality. However, instead of disputing which theory is better, it is possible to integrate the best achievements of both theories and to create one unified, integrated and solid financial theory, which I call the Unified Capital Asset Pricing Model. The integration will lead to better results and to more accurate financial reality description.

The literature suggests to involve only two powers in explanation of securities' prices (*P*). One of them is rational and described by *fundamental* models (*F*). The other power is not rational (*NF*), which is either the *noise* expressed in terms of technical analysis or the *behavioral power*, where psychological biases can affect decision–making process. The unified model assumes that the price of a security should consist of all three powers (rational and non–rational), where noise (*N*) and behavioral (*B*) biases together compose the non–fundamental price (*NF*):

$$P = F + NF,$$

where $$NF = N + B,$$

hence: $$P = F + (N + B).$$

The creation of the Unified Capital Asset Pricing Model is motivated by *unification*, where rational–based and non–rational–based approaches are integrated into one pricing mechanism and by *universality*, allowing use of the same asset pricing mechanism by both rational–based and non–rational–based individuals, meaning in capital asset pricing it is necessary to consider all possible types of investors.

From here the main goal of this PhD thesis is building the model of capital asset pricing, which has a predictive power and is more consistent with real economic data than existing normative and behavioral models. The sub–goals are as follows:

1. Presenting normative and behavioral approaches to asset pricing and comparing them.
2. Describing and comparing empirical findings on non–fundamental component as well as on normative, behavioral and unified models.
3. Proposing and testing the mechanism allowing capital pricing assets, which can be used in investment decision process.
4. Comparing the proposed model to existing models and checking whether it has more predictive power than those models.

The hypotheses are that the unified model has higher predictive power and coefficient stability than technical or sentiment models separately.

To verify the hypotheses the procedure of 4 stages and the data for 2 stock markets of the US and Israel in 2001–2017 was applied. In the 1st stage all necessary variables, goals and hypotheses as well as the analysis procedure and sample definition were introduced. In the 2nd stage the results of models, derived from the principal component analysis were described and compared. The estimation method is *OLS* for all regressions, as acceptable in the literature. Third stage includes integration of all models with relevant fundamental factors and further comparison of estimation results. Finally, the conclusions and final remarks were introduced.

The analysis shows that the unified model demonstrates much better results than fundamental, technical and sentiment models separately. It is even better than alternative model which includes fundamental, technical and sentiment components composed together but not integrated as it appears in the unified model. Those results are stable in the whole examined period on the US and Israeli stock markets.

It was found that technical and sentiment models for both markets demonstrate very similar performance and coefficient patterns, though some lack in the number of sentiment variables exists. The unified model significantly surpasses both models in every compared parameter. The potential alternative model also exhibits good performance, surpassing technical or sentiment models but not the unified model.

Another interesting finding is the similarity in patterns for the US and Israeli stock markets, suggesting that such phenomenon can be cross–boarding. If so, the unified model can be even more universal. For this reason, such phenomenon should be investigated more deeply.

The Unified Capital Asset Pricing Model meets the hypothesizes and goals proposed in this

PhD thesis. Indeed, it constitutes real improvement in capital asset pricing, though in some lower degree than expected. However, it connects existed approaches in attempt to create one unified, universal and solid financial platform.

Streszczenie

Przez dziesięciolecia dyskusje na temat charakteru odchyleń od ceny fundamentalnej (*F*) toczą się albo na gruncie racjonalnym, albo behawioralnym. Dwie główne teorie finansowe – normatywna i behawioralna – istnieją obok siebie, opisując te same zjawiska finansowe w zakresie wyceny aktywów kapitałowych za pomocą różnych modeli, które zaprzeczają sobie nawzajem. Żadna z nich nie ma wystarczających dowodów, aby odrzucić konkurenta, każda ma jednak wystarczające powody na poparcie własnych poglądów. Obie teorie są dobre, ale wydaje się, że nie są wystarczająco dobre. W przeciwnym razie tylko jedna z nich opisywałaby rzeczywistość finansową. Jednakże zamiast spierać się, która z nich jest lepsza, można dokonać próby połączenia najważniejszych osiągnięć obu teorii i w ten sposób dać pole do stworzenia jednego Ujednoliconego Modelu Wyceny Aktywów Kapitałowych. Zintegrowanie najważniejszych osiągnięć obu teorii powinno doprowadzić do lepszych wyników i bardziej precyzyjnego opisu rzeczywistości finansowej.

Literatura przedmiotu sugeruje, że do objaśniania cen papierów wartościowych (*P*) służą tylko dwie siły. Jeden z nich jest racjonalna i opisana przez modele fundamentalne (*F*). Druga jest nieracjonalna (*NF*), i występuje pod postacią *szumu informacyjnego* wyrażanego w kategoriach analizy technicznej lub *zachowań* uczestników rynku, które wynikają z błędów psychologicznych i wpływają na proces decyzyjny. Założeniem ujednoliconego modelu jest wyrażenie w cenie wszystkich trzech sił (racjonalnej i nieracjonalnych), w których szum informacyjny (*N*) i strona behawioralna (*B*) decyzji inwestycyjnych razem składają się na cenę niefundamentalną (*NF*):

$$P = F + NF,$$

gdzie $NF = N + B,$

zatem: $P = F + (N + B).$

Motywacją do stworzenia Ujednoliconego Modelu Wyceny Aktywów Kapitałowych jest *ujednolicenie*, w którym podejście oparte na przesłankach racjonalnych i nieracjonalnych jest zintegrowane w jednym mechanizmie ustalania cen oraz *uniwersalnością*, umożliwiającą stosowanie tego samego mechanizmu wyceny aktywów zarówno przez osoby racjonalne oraz osoby inne niż racjonalne, co oznacza, że w wycenie aktywów kapitałowych konieczne jest rozważenie wszystkich możliwych rodzajów inwestorów.

Biorąc pod uwagę powyższe, głównym celem tej pracy doktorskiej jest budowa modelu wyceny aktywów kapitałowych, który ma moc predykcyjną i jest bardziej zgodny z rzeczywistymi danymi finansowymi niż istniejące modele normatywne i behawioralne. Cele szczegółowe są następujące:

1. Przedstawienie normatywnych i behawioralnych podejść do wyceny aktywów kapitałowych i ich porównanie.
2. Opisywanie i porównywanie wyników badań empirycznych dotyczących elementów innych niż fundamentalne, a także modeli normatywnych, behawioralnych z modelem ujednoliconym.
3. Zaproponowanie i przetestowanie ujednoliconego mechanizmu umożliwiającego wycenę aktywów kapitałowych, który może być wykorzystany w procesie podejmowania decyzji inwestycyjnych.
4. Porównanie zaproponowanego modelu z istniejącymi modelami i sprawdzenie, czy ma on większą moc predykcyjną niż te modele.

W pracy zweryfikowano trzy hipotezy o tym, że ujednolicony model ma wyższą moc predykcyjną i stabilność współczynników niż modele uwzględniające wskaźniki analizy technicznej lub wskaźniki nastrojów rynkowych rozpatrywane oddzielnie.

W celu weryfikacji hipotez zastosowano procedurę badawczą złożoną z 4 etapów i dane dla dwóch giełd papierów wartościowych: w USA i w Izraelu w latach 2001–2017. W pierwszym etapie badania zdefiniowano wszystkie niezbędne zmienne, cele i hipotezy, a także opisano procedurę badania i próbę badawczą. W drugim etapie zaprezentowano i porównano parametry modeli. Metodą estymacji dla wszystkich regresji jest *OLS*, co jest powszechnie akceptowane w literaturze. Trzeci etap badania obejmuje integrację wszystkich modeli z odpowiednimi czynnikami fundamentalnymi i dalsze porównanie wyników estymacji. Na koniec zaprezentowano wnioski i uwagi końcowe.

Analiza przeprowadzona w tej pracy doktorskiej pokazała, że ujednolicony model wyceny aktywów kapitałowych daje znacznie lepsze wyniki niż modele bazujące osobno na czynnikach fundamentalnych, technicznych lub behawioralnych. Jest nawet lepszy niż model alternatywny, który składa się z elementów fundamentalnych, technicznych i behawioralnych złożonych razem, ale niezintegrowanych, tak jak ma to miejsce w modelu ujednoliconym. Wyniki te są stabilne w całym badanym okresie, zarówno na rynku amerykańskim, jak i izraelskim.

Stwierdzono, że modele uwzględniające wskaźniki techniczne i nastrojów rynkowych na obu rynkach wykazują bardzo podobne wyniki i wartości współczynników, chociaż niektóre z nich nie mają wystarczającej liczby zmiennych sentymentalnych. Model ujednolicony znacznie przewyższa oba modele pod względem każdego porównywanego parametru. Model alternatywny także cechuje się dobrą jakością i jest lepszy od modelu technicznego i behawioralnego, ale nie od modelu ujednoliconego.

Kolejnym interesującym rezultatem jest podobieństwo wyników dla rynków amerykańskiego i izraelskiego, co sugeruje, że zaproponowany model może być wykorzystywany w badaniach o zasięgu międzynarodowym. Jeśli tak, model ujednolicony może być jeszcze bardziej uniwersalny. Jednakże, aby mieć pewność, czy tak faktycznie jest, należy przeprowadzić dalsze badania.

Ujednolicony Model Wyceny Aktywów Kapitałowych spełnia założenia zaproponowane w tej pracy doktorskiej. Faktycznie dokładniej niż dotychczasowe modele wycenia on aktywa kapitałowe, choć w mniejszym stopniu niż oczekiwano. Łączy jednak istniejące podejścia i tworzy jedną ujednoliconą, uniwersalną i solidną platformę finansową do wyceny aktywów kapitałowych.